Strongly Interacting Dark Sectors, Dark Matter & Phase Transitions

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University of Sussex, February 10, 2025

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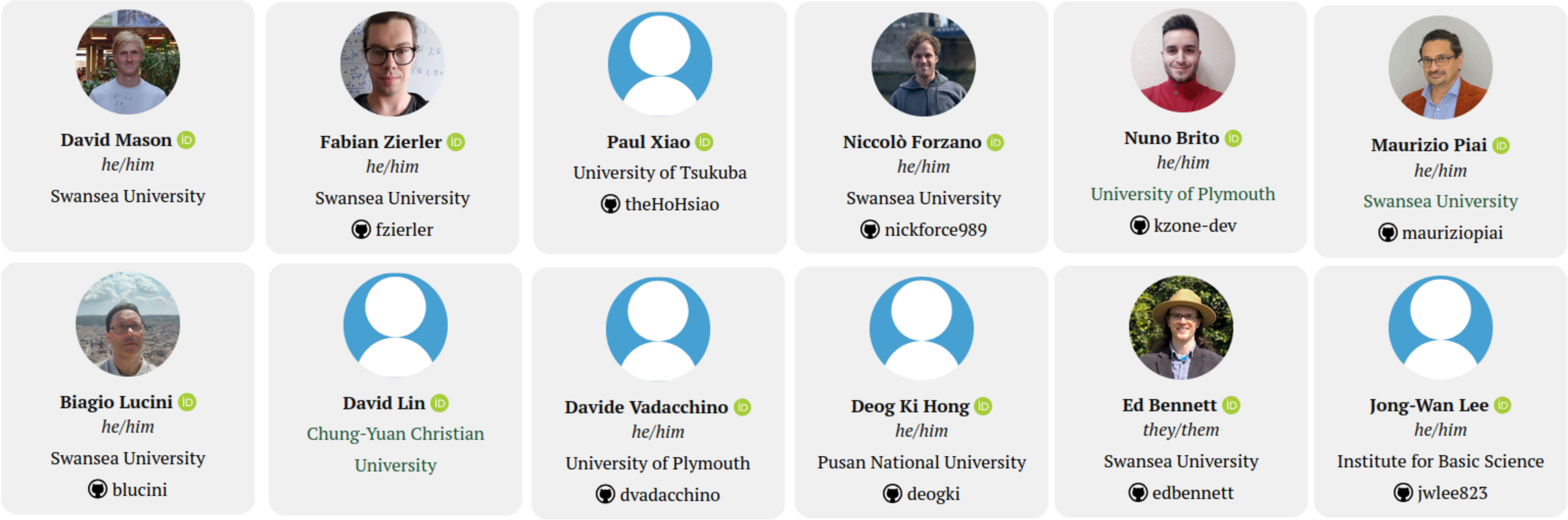
TELOS Collaboration

telos-collaboration.github.io/

Theoretical **E**xplorations on the **L**attice with **O**rthogonal and **S**ymplectic groups.



Overview

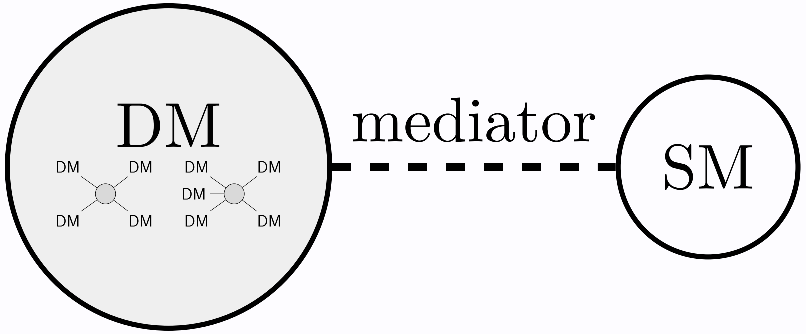
- **Strongly interacting models beyond the Standard Model**
  - Pure Yang-Mills theories: Dark Glueballs
  - With fermions: Composite Dark Matter/composite Higgs+top
- **1st order phase transitions in those models**
  - Pure Yang-Mills: Mostly 1st order
  - With Fermions: Columbia plot and its generalizations
- **First-order transitions on the lattice**
  - Issues with importance sampling algorithms
  - Density-of-states approaches

Strongly interacting BSM models

1. Dark Matter Models

Strongly Interacting Gauge Theories in DM Models

- Pure gauge: Dark glueballs only possibility
- With fermions: Global symmetries make DM stable
- **Mediator typically required**: Dark sector coupled to SM



$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_f(i\not{D} + m_f)\psi_f$$

- Non-vanishing self-scattering cross-sections: $\langle v\sigma_{\mathrm{DM}\to\mathrm{DM}}\rangle \neq 0$
- Relic density driven by strong processes

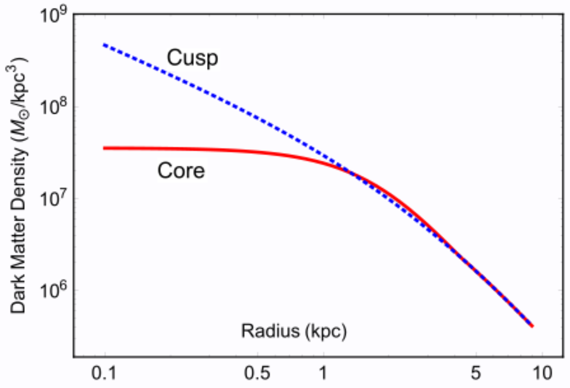
Gauge-fermion theories: Global symmetries

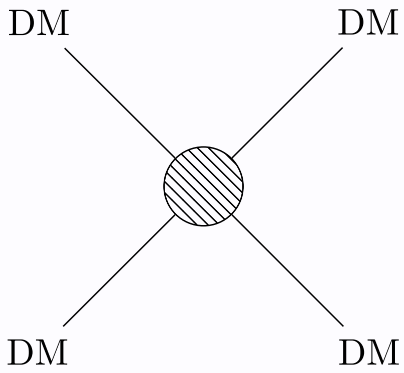
- in QCD with massless fermions: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$
- Different breaking patterns $G \to H$ for fermion irreps

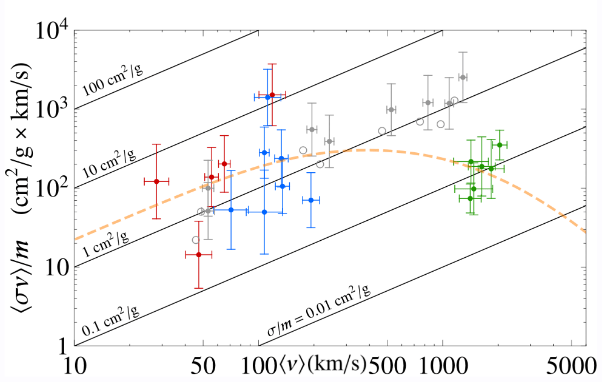
	complex	**pseudo-real**	**real**
	$U(N_f) \times U(N_f)$	$U(2N_f)$	$U(2N_f)$
↓ axial anomaly: $m_i = 0$	↓	↓	↓
	$SU(N_f)_L \times SU(N_f)_R \times U(1)_B$	$SU(2N_f)$	$SU(2N_f)$
↓ chiral symmetry breaking: $m_i = 0$ *and/or* explicit breaking: $m_i = m_j \neq 0$	↓	↓	↓
	$SU(N_f)_V \times U(1)_B$	$Sp(2N_f)$	$SO(2N_f)$
↓ strong isospin breaking: $m_i \neq m_j$	↓	↓	↓
	$U(1)^{N_f}$	$SU(2)^{N_f}$	$SO(2)^{N_f}$

Dark Matter properties

- DM self-interaction phenomenologically allowed[1] and potentially relevant for small-scale structure problems
  - non-vanishing scattering cross-sections $\sigma_{\mathrm{2DM}\to\mathrm{2DM}}$
  - velocity dependence of $\sigma_{\mathrm{2DM}\to\mathrm{2DM}}$ preferred







QCD-like Dark Matter can those provide self-interactions!

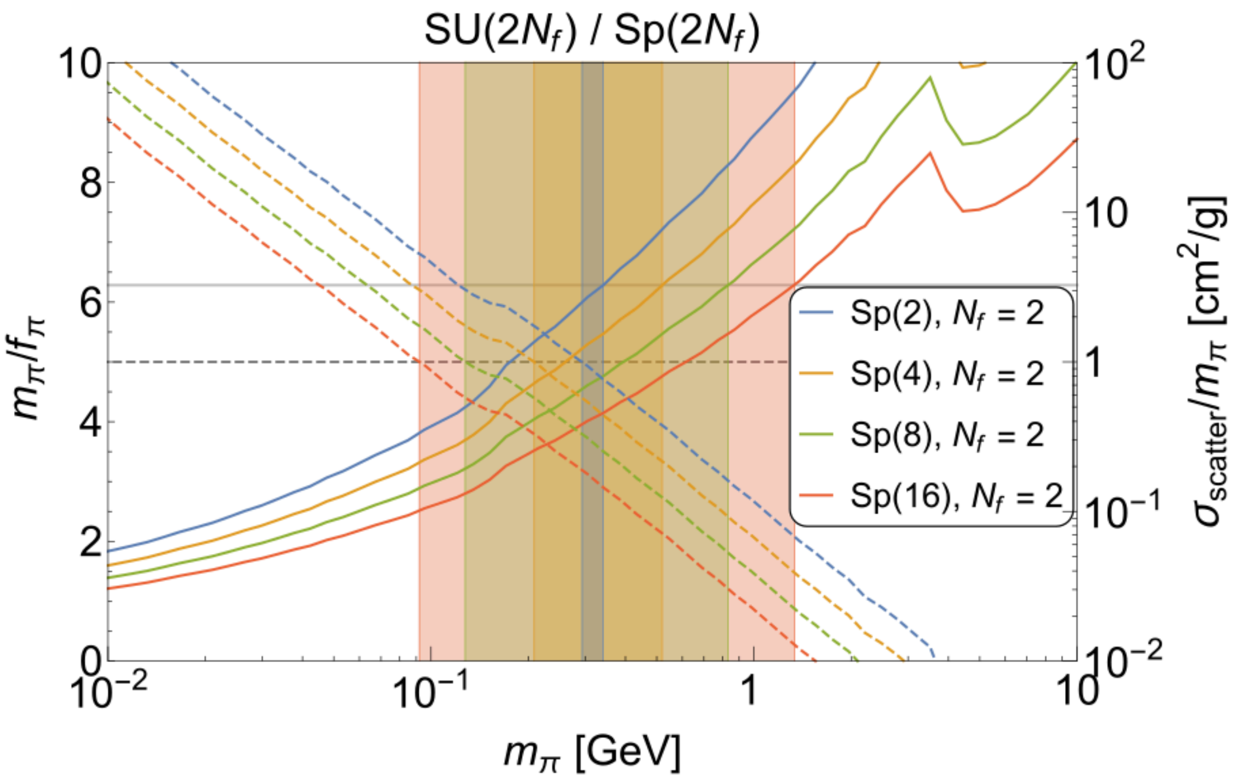
[1] Hochberg et. al. [1402.5143][1411.3727]

[2] Wess, Zumino(Phys. Lett. B 1971), Witten (Nucl. Phys. B 1983)

Example model:

Strongly **I**nteracting **M**assive **P**articles

- In ChiPT: Wess-Zumino-Witten
- Depletion via $3\mathrm{DM} \rightarrow 2\mathrm{DM}$ [1]
  - same as $KK \rightarrow 3\pi$ in QCD [2]
  - LO ChiPT matches relic density at $m_\pi \approx \mathcal{O}(100)\mathrm{MeV}$



Strongly interacting BSM models

2. Composite Higgs Models

(and partial top compositeness)

Here: Only asymptotically free gauge-fermion theories

Composite Higgs Scenarios with Partial Top Compositeness

- Are much more constrained by electroweak phenomenology
  - Must accommodate SM: $H \supset G_{\text{custodial}} \supset G_{\text{SM}}$
  - Must contain a Higgs doublet, and a fermionic trilinear
  - retain asymptotic freedom
  - want to avoid IR conformal theories

`Requires more than one fermion irrep:`

$$\mathcal{L}_{\mathrm{CH}} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\mathrm{Q}}^{\mathrm{i}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right)\mathrm{Q}^{\mathrm{i}} + \bar{\Psi}^{\mathrm{j}}\left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right)\Psi^{\mathrm{j}}$$

table taken from Feretti [1604.06467], see also Ferretti,Karateev [1312.5330]

Composite Higgs + Top Realisations as classified by Ferretti and Karateev

G_{HC}	ψ	χ	Restrictions	G/H
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$N_{\mathrm{HC}}=7,9$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=7,9$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5\times\mathbf{A}_2$	$6\times\mathbf{F}$	$2N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)}\,\mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$5\times\mathbf{A}_2$	$3\times(\mathbf{F},\overline{\mathbf{F}})$	$N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)}\,\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5\times\mathbf{F}$	$3\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$N_{\mathrm{HC}}=10$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$4\times\mathbf{F}$	$6\times\mathbf{A}_2$	$2N_{\mathrm{HC}}=4$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$4\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=11$	
$\mathrm{SO}(N_{\mathrm{HC}})$	$4\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$6\times\mathbf{F}$	$N_{\mathrm{HC}}=10$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\,\frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}\,\mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$6\times\mathbf{A}_2$	$N_{\mathrm{HC}}=4$	
$\mathrm{SU}(N_{\mathrm{HC}})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{A}_2,\overline{\mathbf{A}}_2)$	$N_{\mathrm{HC}}=5,6$	$\frac{\mathrm{SU}(4)\times\mathrm{SU}(4)'}{\mathrm{SU}(4)_D}\,\frac{\mathrm{SU}(3)\times\mathrm{SU}(3)'}{\mathrm{SU}(3)_D}\,\mathrm{U}(1)$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

2. Gauge theories with 1st order transitions

see e.g. Wiese et.al. [hep-lat/0312022]

First-order phase transitions in gauge theories

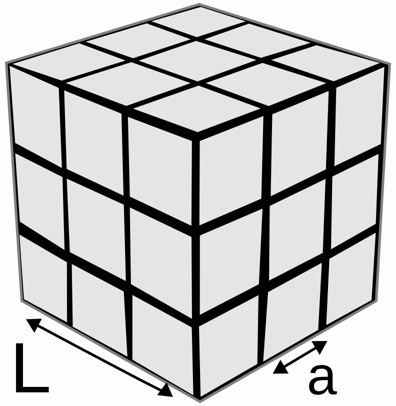
- Finite temperature confinement-deconfinement transitions
- Oder parameter: Polyakov loop
- **Typically first order**: exception $SU(2)$, $SU(3)$ weakly first oder
- Driven by the size of the group: Also $Sp(2N > 4)$ is first order
  - Larger $N_c \to$ stronger PT
- With sufficiently heavy fermions: Still a first oder transition

Intermission: Lattice setup

- Euclidean action S on hypercubic lattice

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]} O[A_\mu, \psi, \bar{\psi}]$$

- Lattice regulator: finite spacing a (UV), finite extent $L = aN_s$ (IR)



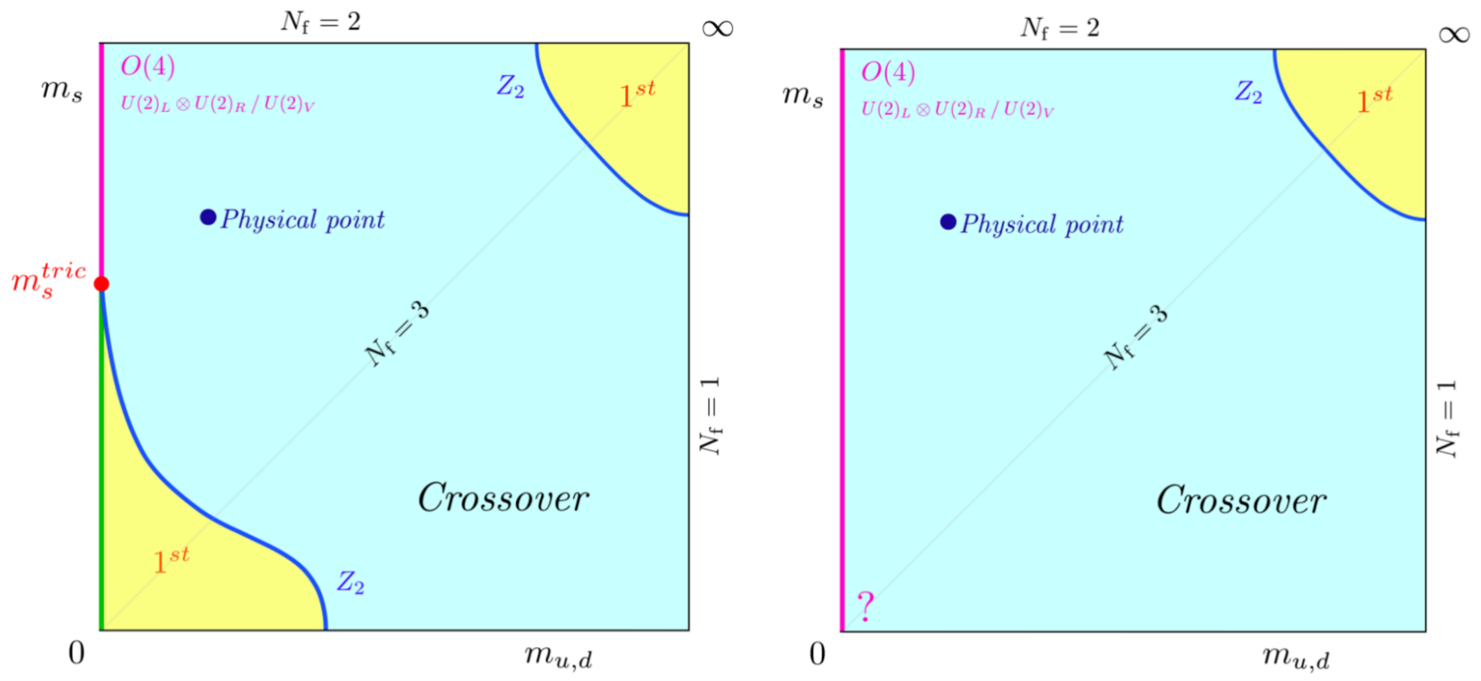
- Calculate observable $\langle O \rangle$ on finite lattice
- Extrapolate to the continuum: $a \to 0$, $L \to \infty$

see e.g. Cuteri et.al. [2009.14033] [2107.12739] Kaiser, Philipsen [2212.14461] Klinger et.al. [2501.19251]

First-order PTs in gauge-fermion theories

The Columbia plot: Strong lattice spacing a-dependence

(Phase diagram for three fermion flavours)

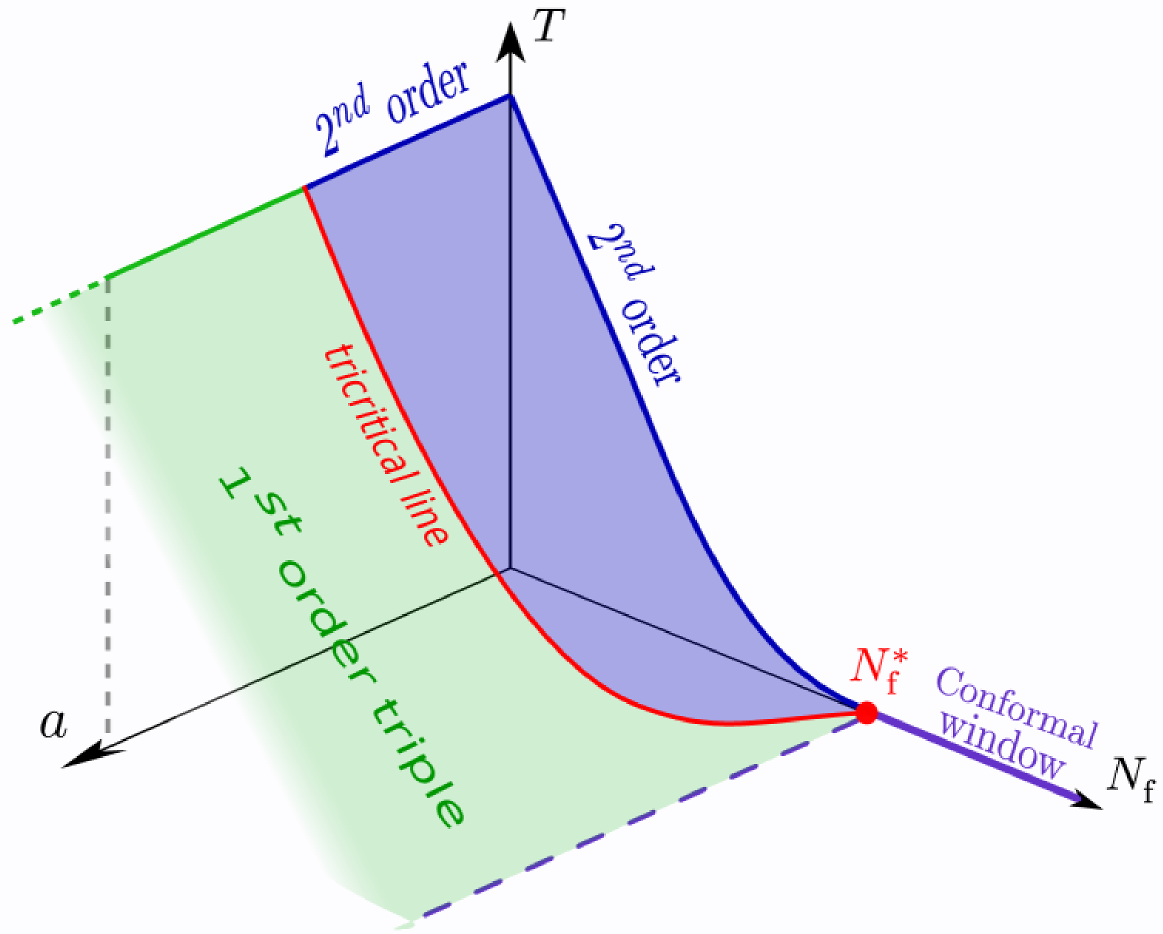


coarse lattice

fine lattice

e.g. Cuteri et.al. [2009.14033] [2107.12739] Kaiser, Philipsen [2212.14461] Klinger et.al. [2501.19251]

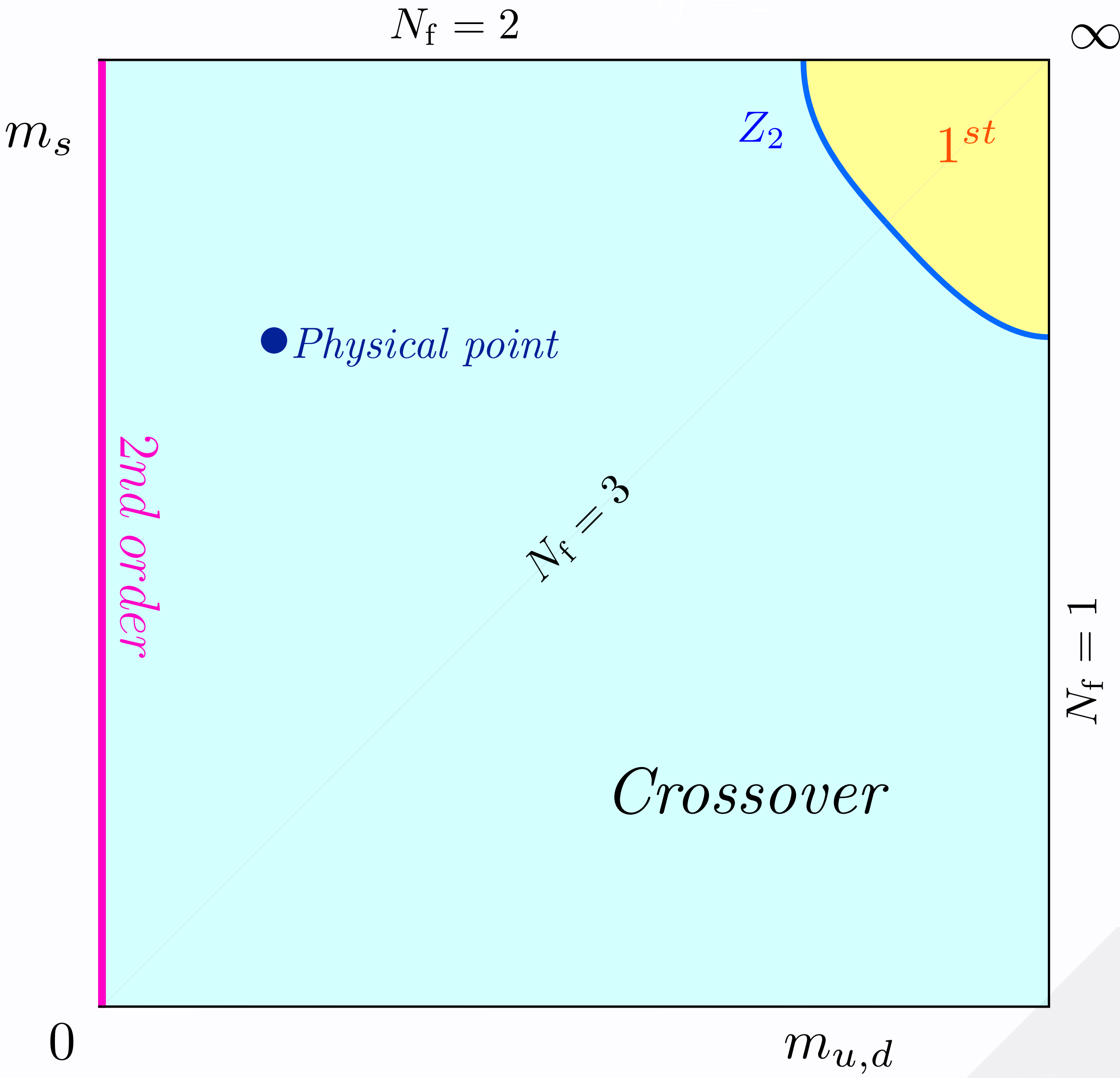
Generalized large-N_f Columbia plot



- here: mass extrapolated to chiral limit $m = 0$
- 1st order phase transition observed for all N_f studied
- However, no transition survives the continuum limit

Summary: Where to expect 1st order transitions?

- Pure gauge theory with $N_c > 2$
- Deconfinement transitions for heavy fermions
  - Upper-right corner of Columbia plot
- Probably no 1st order chiral phase transition



First-order transitions on the lattice

Finite temperatures on the lattice

- Principle: Trade time for temperature T
- Lattice of size $(aN_s)^3 \times (aN_t)$ corresponds to $T = 1/aN_t$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] O[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]}$$

- Fermions can be integrated analytically

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu] Z[A_\mu] O[A_\mu] e^{-S_G[A_\mu]}$$

- We need to evaluate this integral
- **Monte-Carlo methods**:
  Sample integral for some gauge field configurations $\{A_\mu^n\}$

Generating configurations: Importance sampling

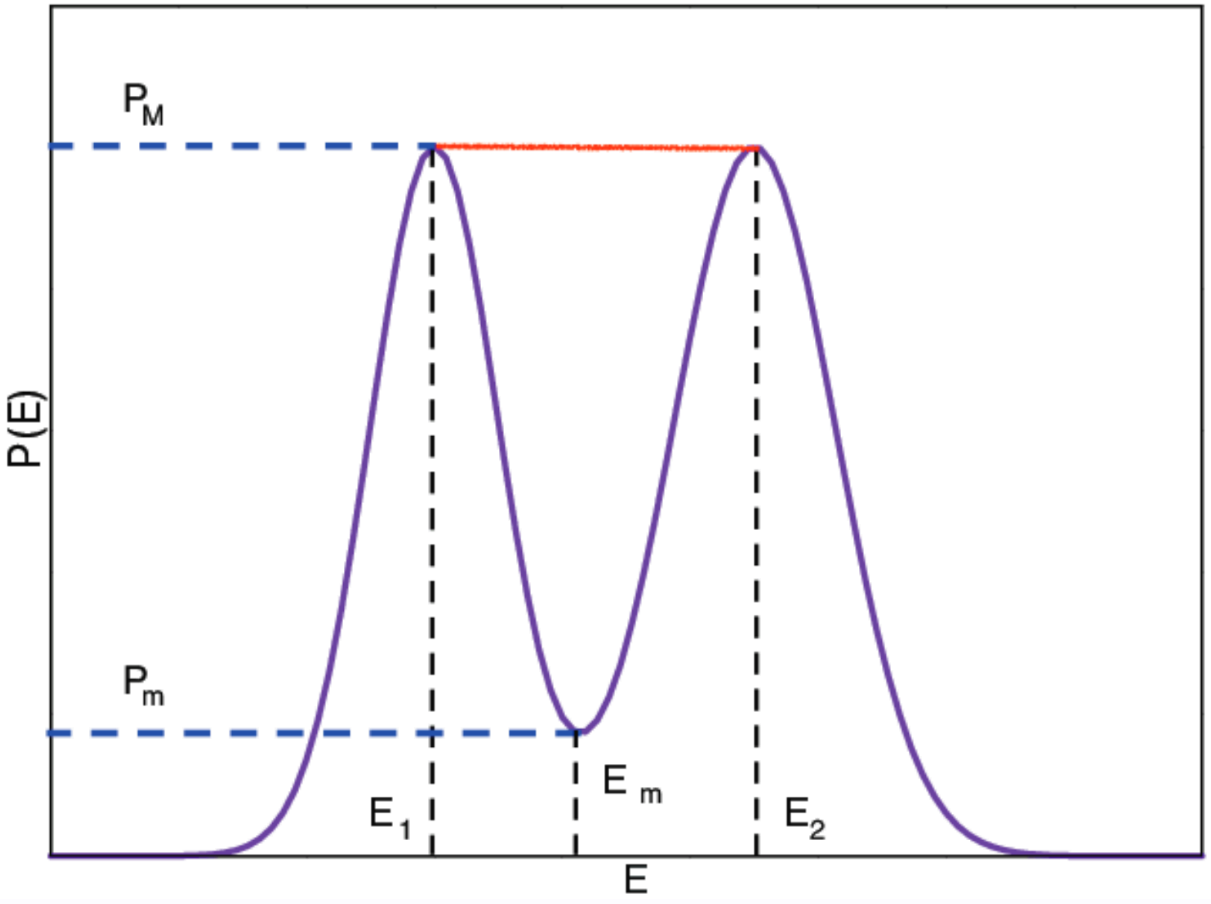
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu] Z[A_\mu] O[A_\mu] e^{-S_G[A_\mu]}$$

- generate a Markov chain: $A_\mu^{(1)} \to A_\mu^{(2)} \to A_\mu^{(3)} \to \cdots \to A_\mu^{(n)} \to \ldots$
- update so that for $n \to \infty$ the $A_\mu^{(n)}$ are distributed according to

$$\mathrm{d}P(A_\mu) = \frac{1}{Z} \mathcal{D}[A_\mu] Z[A_\mu] e^{-S_G[A_\mu]}$$

- then observables are calculated by

$$\langle O \rangle = \frac{1}{N_{\mathrm{conf}}} \sum_{i=1}^{N_{\mathrm{conf}}} O[A_\mu^{(i)}]$$



1st order transitions & importance sampling

- The probability $\mathrm{d}P(A_\mu)$ has two peaks (phase coexistence)
- Markov chain tends to get stuck in one peak
- Situation worsens as the continuum is approached

see e.g. Langfeld et.al. [1204.3243] [1509.08391]

Density-of-states approaches

- Alternative microcanoical approach

$$Z = \int D[\phi] e^{\beta S[\phi]} = \int \mathrm{d}E \rho(E) e^{\beta S[\phi]}$$

$$\rho(E) = \int D[\phi] \delta(S[\phi] - E)$$

- If we were to know $\rho(E)$ we can reconstruct every observable that only depends on the energy E
- By determining $\rho(E)$ for fixed E we circumvent the problems faced by importance sampling

see e.g. Langfeld et.al. [1204.3243] [1509.08391]

Determining the d.o.s.: Linear Logarithmic Relaxation

$$\rho(E) = \rho(0) \exp\left[-\int_0^E \mathrm{d}\tilde{E}\, a(\tilde{E})\right]$$

- In practice, we determine $a(E)$ for discrete values E_k of width δ
- Restrict action $S[\phi]$ to energy interval for given $a(E_k)$

$$\langle\langle O \rangle\rangle (a) = \int D[\phi] O[\phi] W(E_k, \delta) \exp\left[a \cdot (S[\phi] - E_k)\right]$$

- Can be calculated on the lattice
- The correct logarithmic density-of-states $a(E)$ fulfills

$$\langle\langle S[\phi] - E_k \rangle\rangle (a) = 0$$

- Can be solved iteratively!

Robbins-Monro solution

$$\langle\langle S[\phi] - E_k \rangle\rangle(a) = 0$$

- A stochastic variation of the Newton-Raphson method

$$a_{n+1} = a_n - c_n \langle\langle S[\phi] - E_k \rangle\rangle(a_n)$$
$$\sum_n c_n^2 \to \text{finite} \quad \sum_n c_n \to \infty$$

- Whole setup for density-of-states: Start with initial a_0
  - Update with Robbins-Monro n times and obtain a_n (the dos)
  - Repeat N times to obtain $\left\{a_n^{(i)}\right\}$
  - Measure observables N times to obtain an error estimate

Observables in the LLR approach

- Operators that are functions of the energy $S[\phi]$

$$\langle O(S[\phi])\rangle = \frac{\int D[\phi] O(S[\phi]) e^{\beta S[\phi]}}{\int D[\phi] e^{\beta S[\phi]}} = \frac{\int \mathrm{d}E\ \rho(E) O(E) e^{\beta E}}{\int \mathrm{d}E\ \rho(E) e^{\beta E}}$$

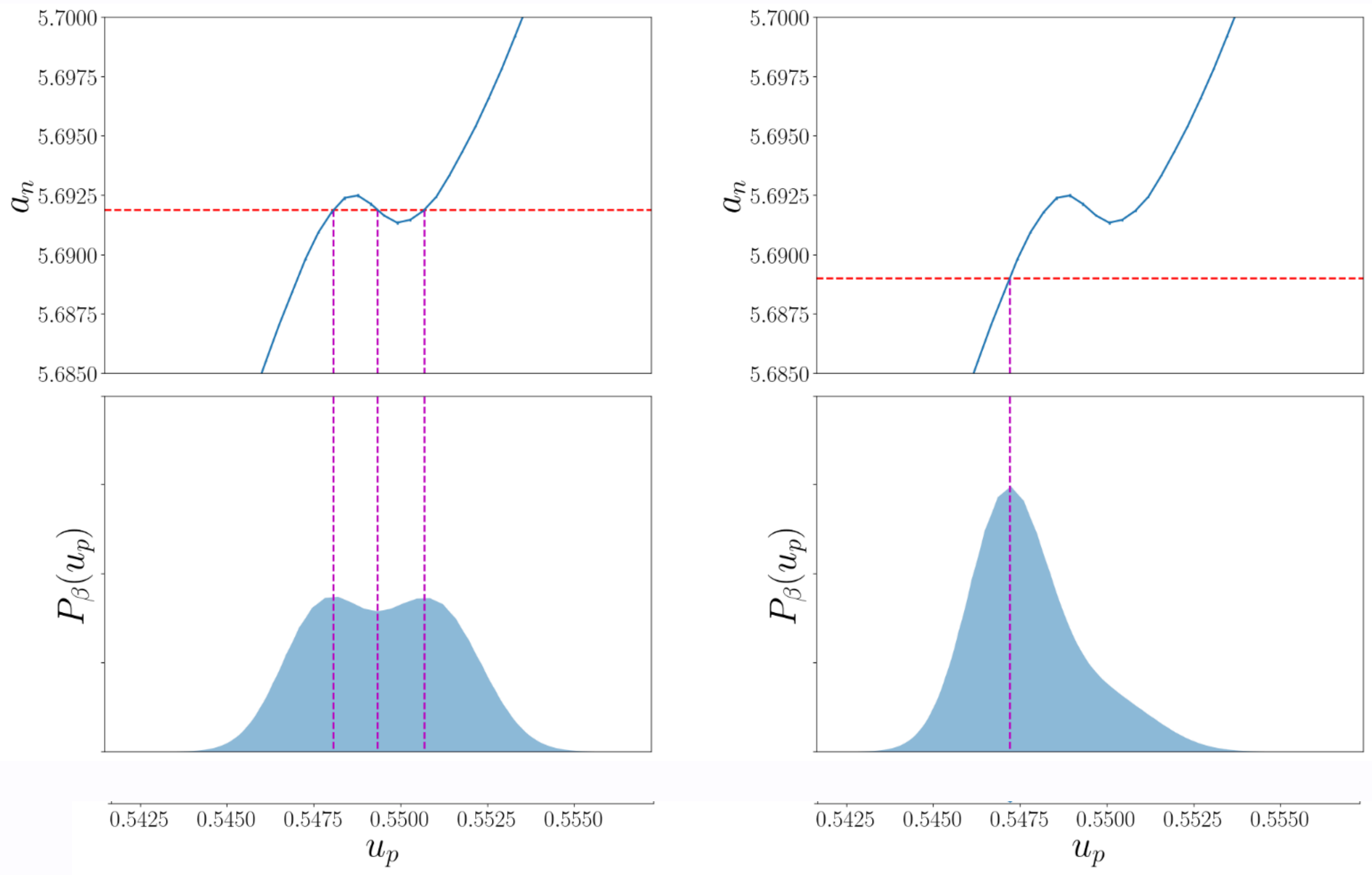
- It can be shown that other observables are also accessible

$$\langle O[\phi]\rangle = \frac{\int \mathrm{d}E\ \rho(E) \langle\langle O\rangle\rangle(a(E)) e^{\beta E}}{\int \mathrm{d}E\ \rho(E) e^{\beta E}}$$

see e.g. Lucini et.al. [2305.07463] Bennett et.al. [2409.19426] David Mason (Thesis - Swansea University)

Some results in pure gauge theory I

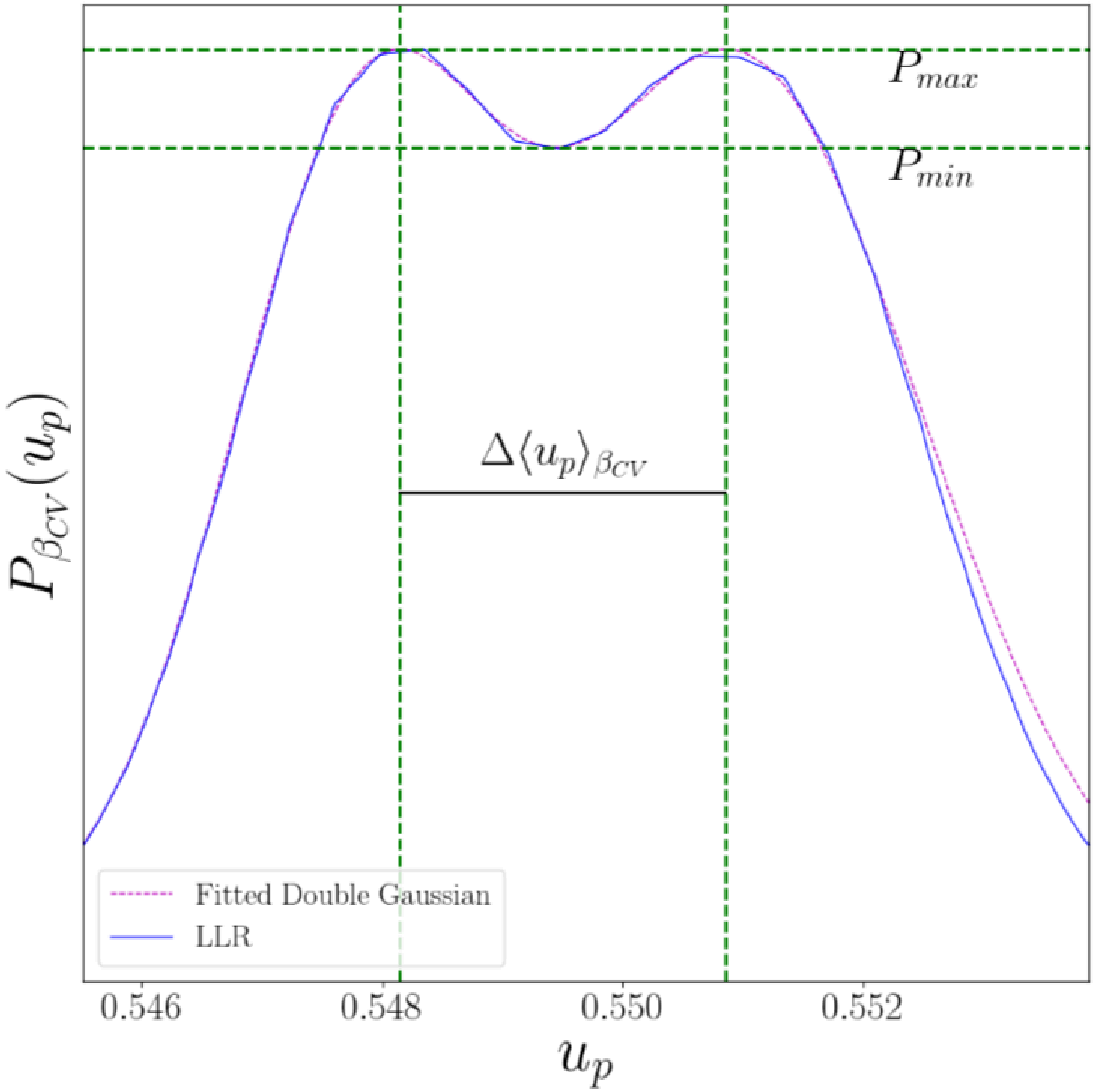
in our lattice discretization $S[\phi] = \frac{6V}{a^4}(1 - u_p[\phi])$



Lucini et.al. [2305.07463] Bennett et.al. [2409.19426]

Relation to latent heat and surface tensions

David Mason (Thesis - Swansea University)



- Difference in plaquette (energy) gives latent heat L_h

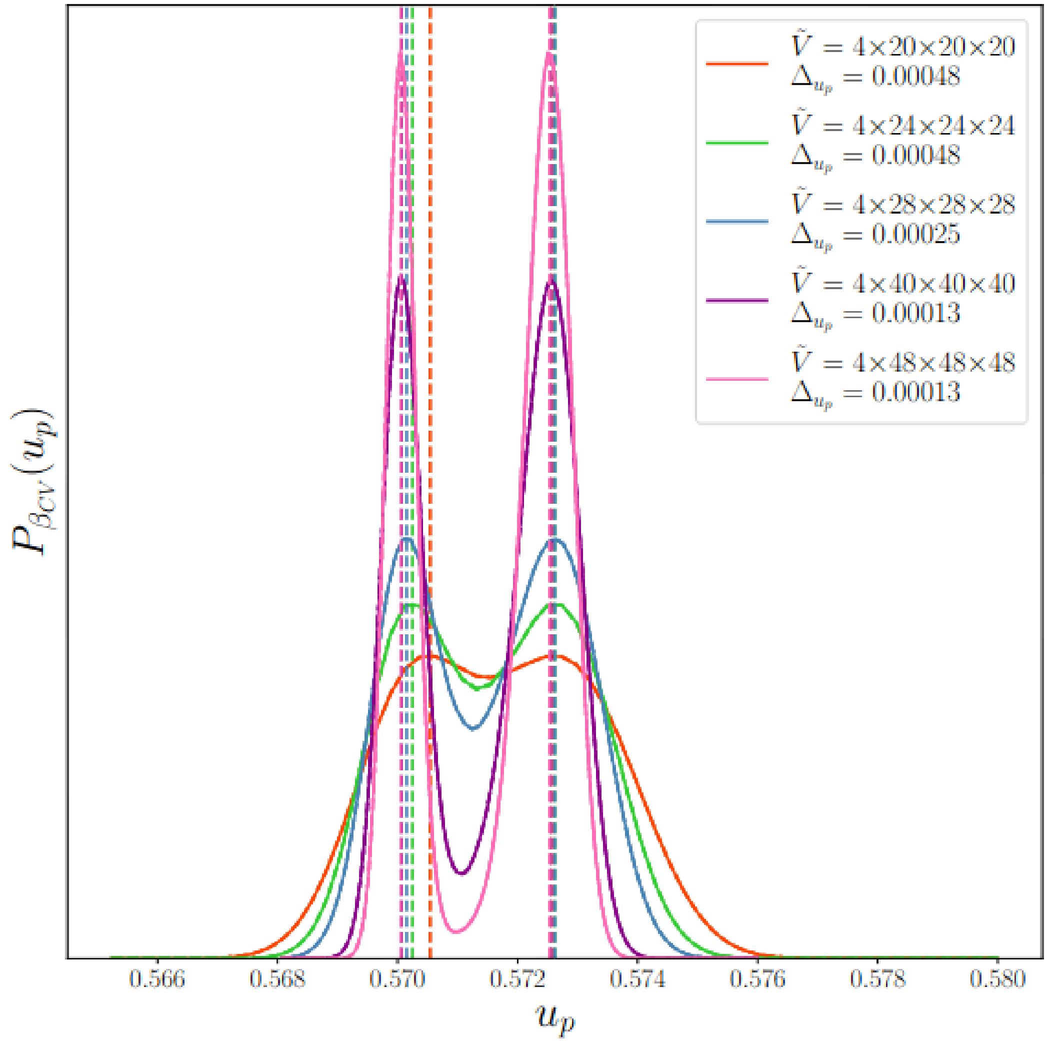
$$\Delta\langle u_p\rangle_{\beta_c} \propto L_h/T_c^4$$

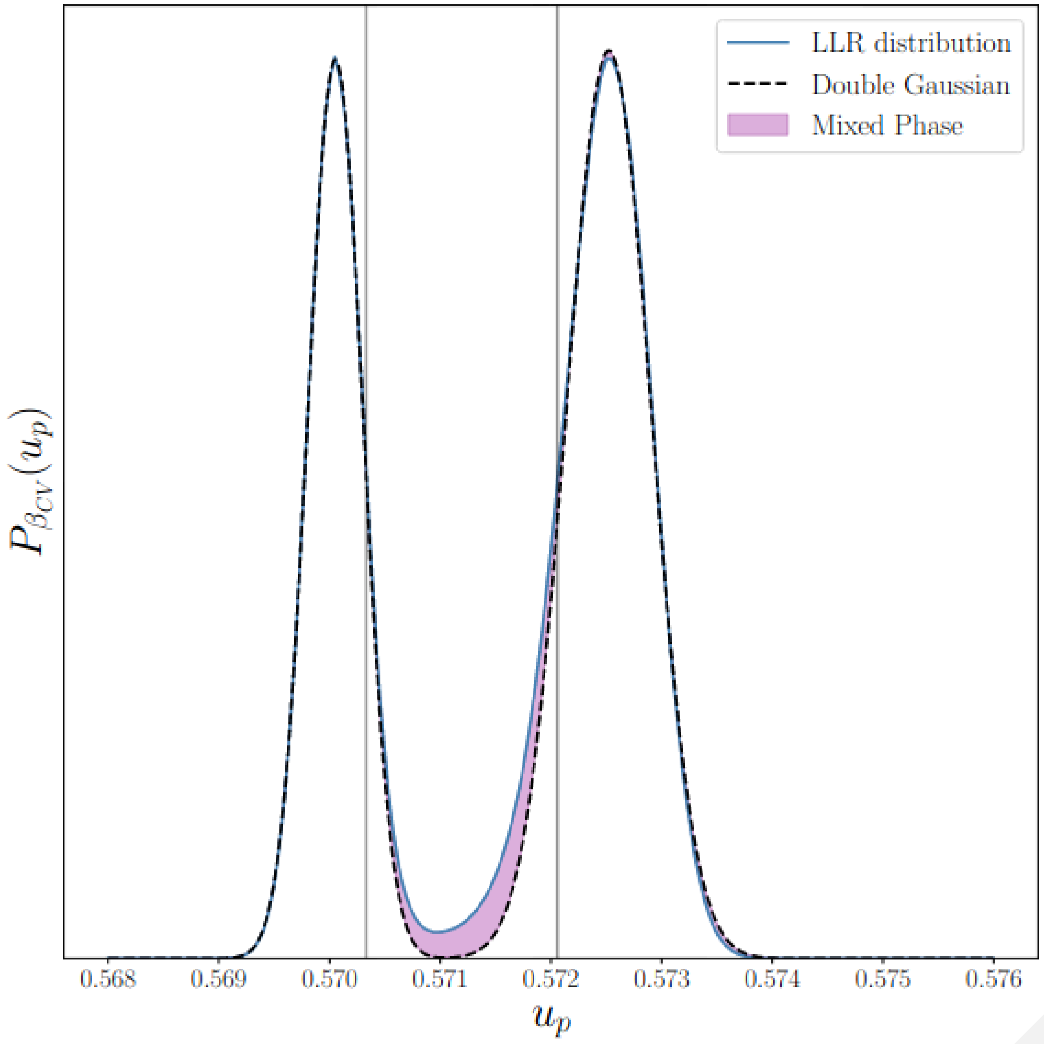
- Peaks of the probability distribution relate to surface tension σ_{cd}

$$\frac{P_{\min}}{P_{\max}} \propto \alpha_1 \exp\left(-\alpha_2\sigma_{cd}/T_c^3\right)$$

see e.g. Lucini et.al. [2305.07463] Bennett et.al. [2409.19426] David Mason (Thesis - Swansea University)

Lattice volume dependence

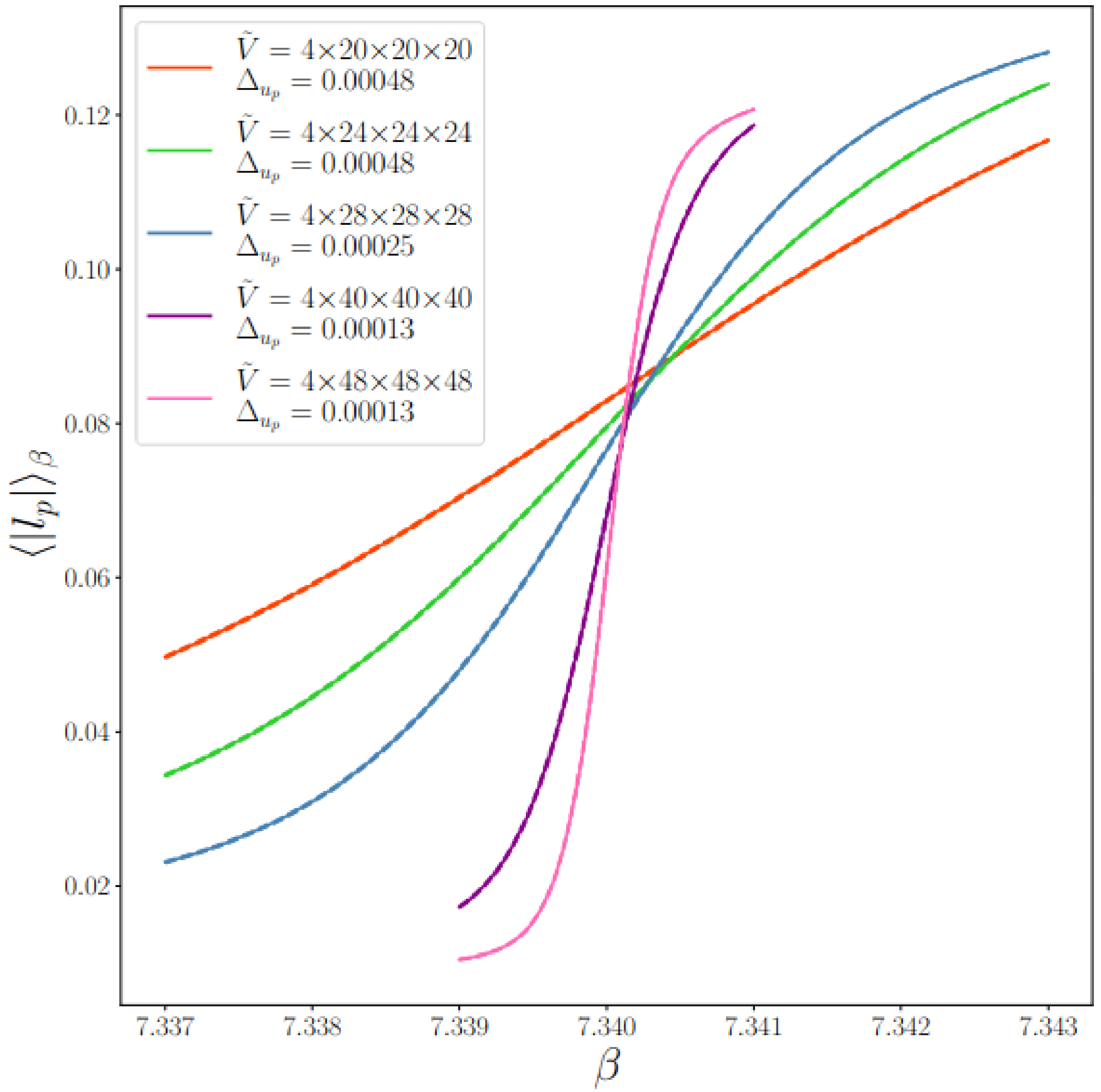




Lucini et.al. [2305.07463] Bennett et.al. [2409.19426]

David Mason (Thesis - Swansea University)

The Polyakov loop



- order parameter for phase transitions
- measured via double-bracket evaluation on restricted energy intervals
- similar strong volume dependence

Summary

- Dark/composite sector are interesting BSM candidates
- First order transitions occur for heavy fermions and in pure gauge
- Standard Lattice techniques struggle with 1st order transitions
- LLR provides an alternative approach to perform first-principles lattice calculations!

Thank you

Back-up slides

Other densities-of-state: Polyakov Loop Potential $V(q)$

- following Langfeld/Pawlowski [1307.0455] it is

$$Z[j] = \int D[\phi] \exp\{S[\phi] + jp[\phi]\}$$

$$V(q) = \frac{T}{\Lambda_3}(jq - \log Z[j]) \qquad q = \frac{\mathrm{d}}{\mathrm{d}j} \log Z[j]$$

- introduce density-of-states of the Polyakov loop

$$\rho(p) = \int D[\phi] \delta(p[\phi] - q) \exp\{S[\phi]\}$$

- Obtain $Z[j]$ and $q[j]$ using this dots

$$q[j] = \frac{1}{Z[j]} \int \mathrm{d}\tilde{\mathrm{q}} \rho(\tilde{\mathrm{q}}) \tilde{\mathrm{q}} \mathrm{e}^{\mathrm{j}\tilde{\mathrm{q}}}$$

Flavour symmetry

- Higher symmetry than QCD-like theories
- Mixing of left- and right-handed Weyl components

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \qquad \begin{matrix} C \ldots \text{charge conj.} \\ S \ldots \text{colour matrix} \end{matrix}$$

$$\mathcal{L}_{\mathrm{DM}} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right)$$

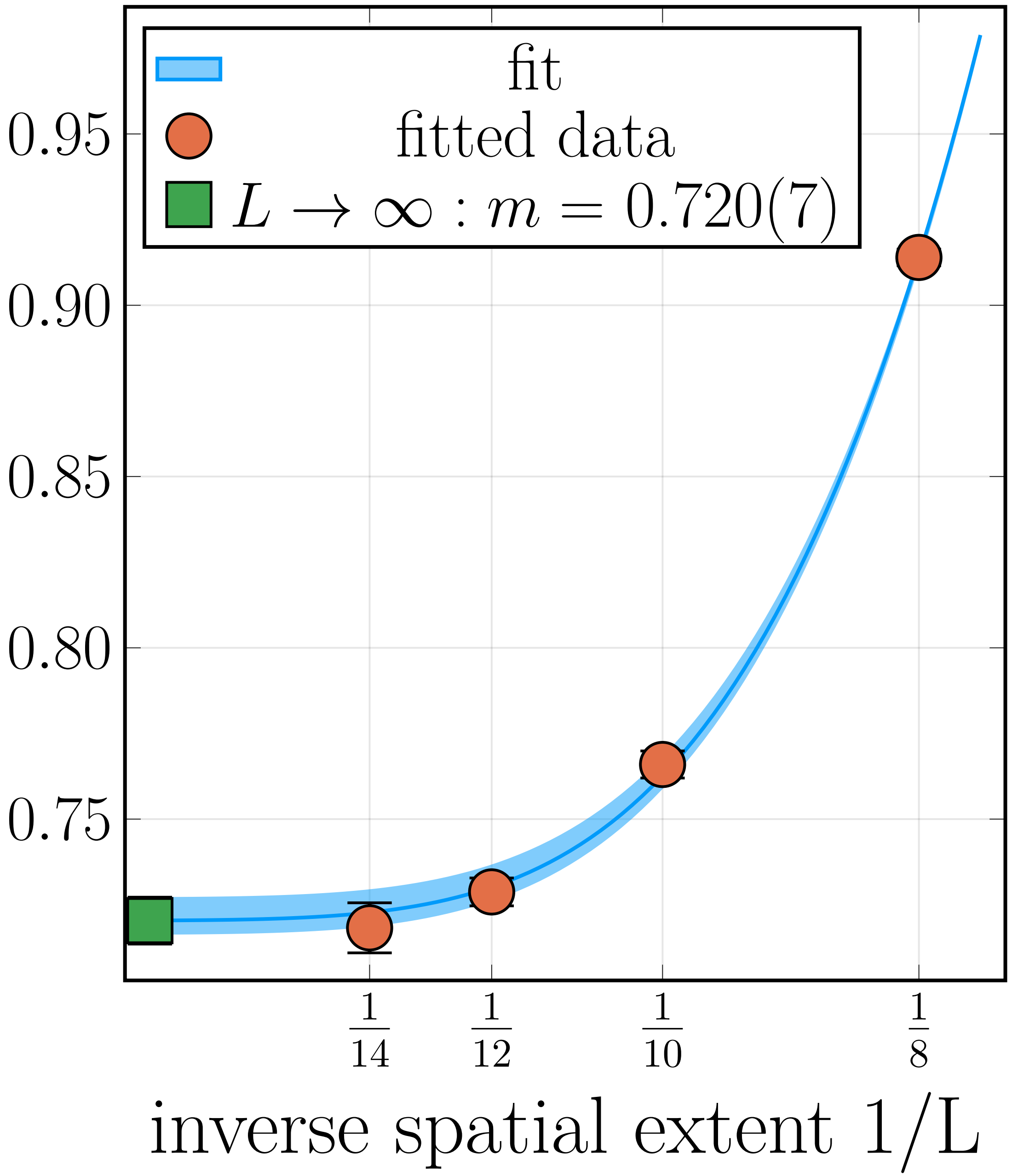
- Mass matrix M proportional to symplectic invariant tensor
- generators τ_a in fundamental : $S\tau_a S = -\tau_a^T$

Dark Matter interaction with SM: Freeze out

- Thermal interactions in early universe
- Assume $\mathrm{SM} \rightleftharpoons \mathrm{DM}$

1. Early universe: equilibrium
2. $\mathrm{SM} \to \mathrm{DM}$ kinematically suppressed
   DM number drops*
3. Not enough DM for more annihilation
   DM number stays constant $\Rightarrow$ *freeze-out* [1]

am(ρ^N): β=6.9 (am_u^0, am_d^0) = (-0.90,-0.89)



[1] Kulkarni et. al. [2202.05191]

Hadronic multiplets of $Sp(4)_c : N_f = 1 + 1$

- Global flavour symmetry breaks: $Sp(4)_F \to SU(2) \times SU(2)$ [1]

  $\mathbf{10} \to \mathbf{6}$ (unflavoured) $+ \mathbf{4}$ (flavoured)

  $\mathbf{5} \to \mathbf{1}$ (unflavoured) $+ \mathbf{4}$ (flavoured)

- *Phenomenological consequences*

  $\Rightarrow$ one π is a singlet $\to$ not protected by symmetry

  $\Rightarrow$ no vector singlets: no mixing of vector mediators (without further symmetry breaking)

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Singlets are particularly interasting for BSM phenomenology!
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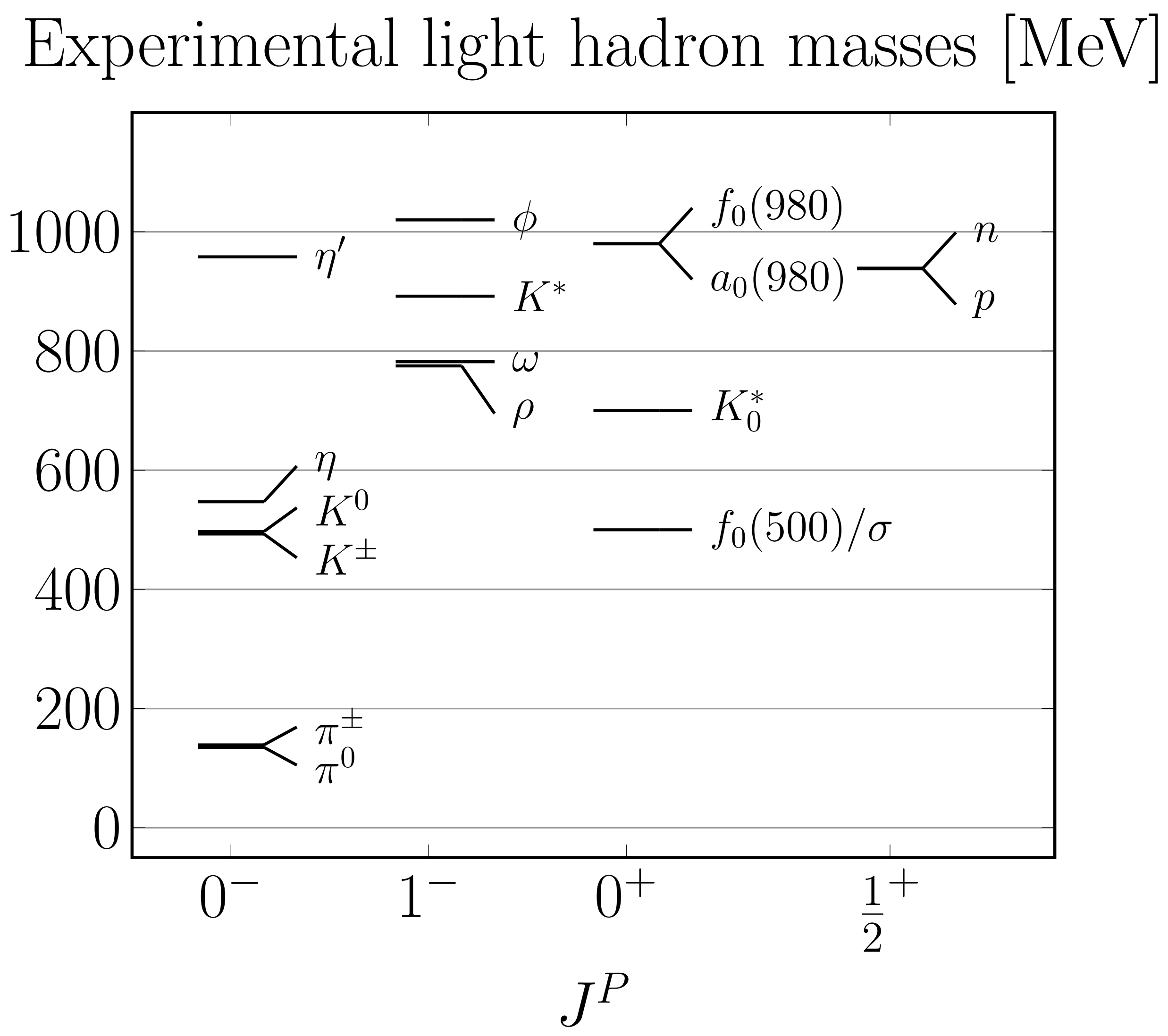
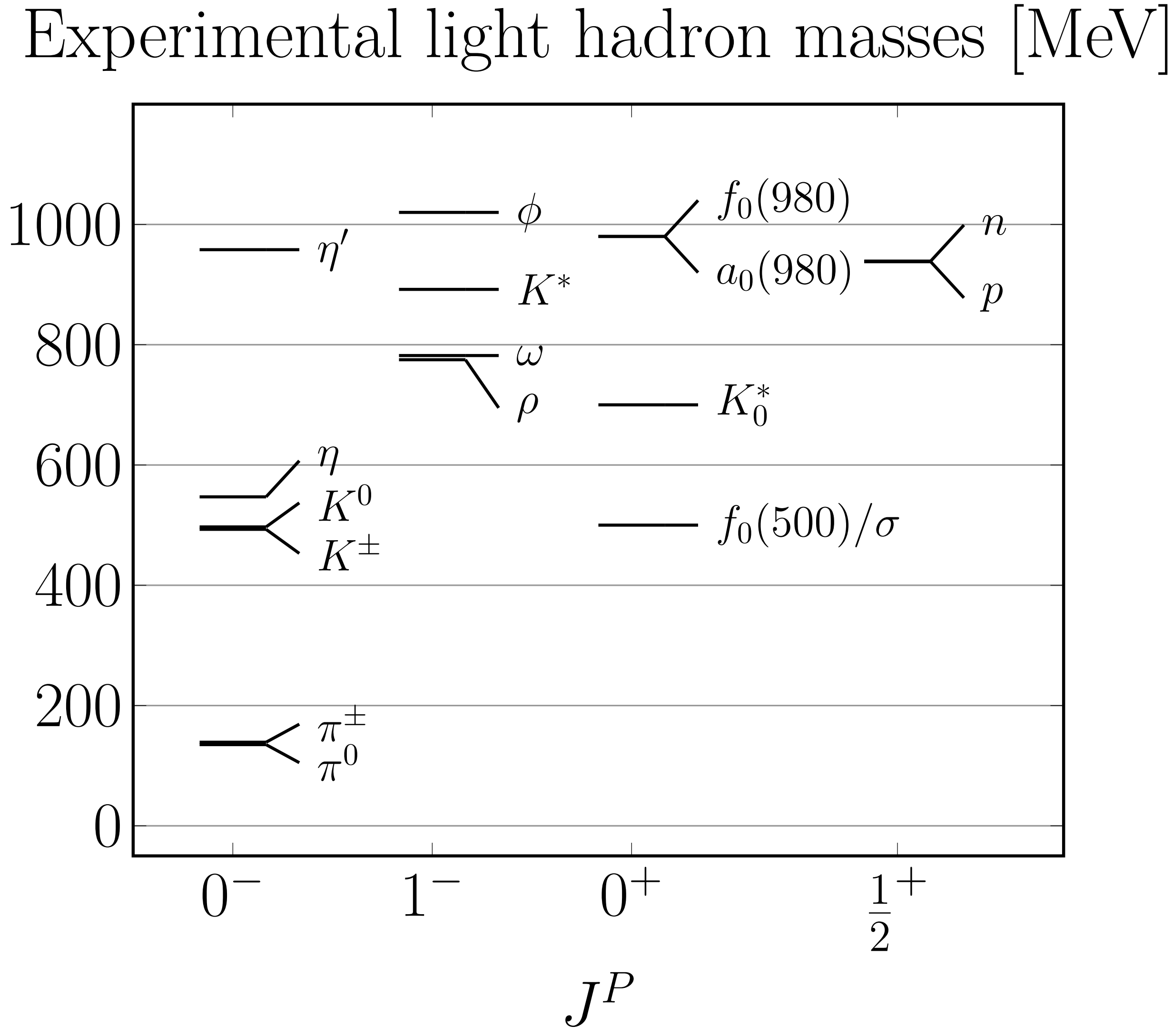


QCD Spectrum

- π, K, η light: pseudo-Goldstones
- Vectors and scalars light
- Light and broad 0^+ singlet f_0/σ
- Heavy 0^- singlet η'
  $\Rightarrow U(1)_A$ anomalously broken

Mesonic multiplets in QCD



- Approximate $SU(2)_F$, $SU(3)_F$
- Mesons in irreps:
  - $SU(2)_F$: triplets and singlets
  - $SU(3)_F$: octets and singlets

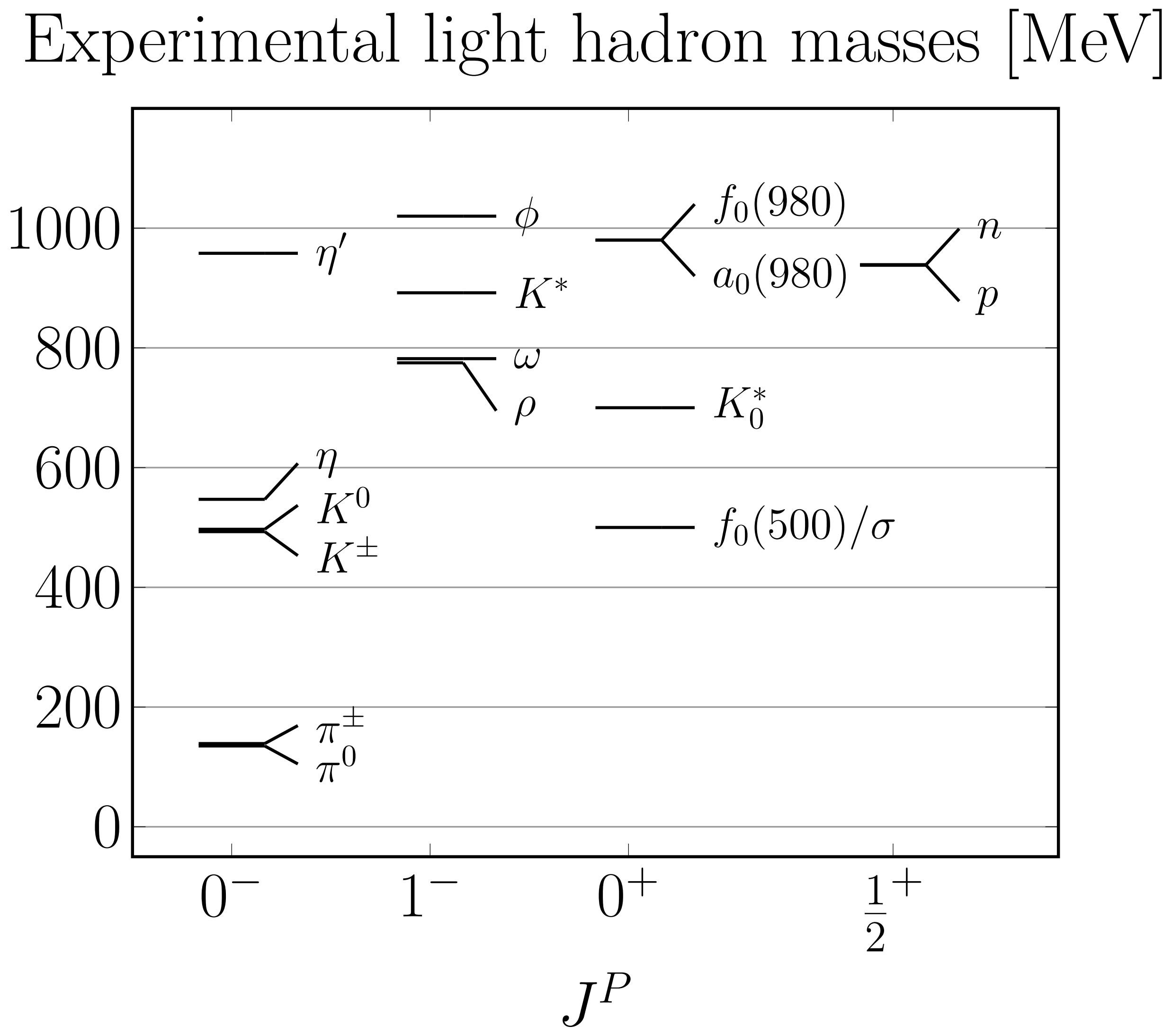
$$\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$$

$$\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{1}$$

$$\text{octets} : (\pi, K, \eta), (\rho, K^*, \omega), \ldots$$

$$\text{triplets} : (\pi^\pm, \pi^0), (\rho^\pm, \rho^0), \ldots$$

$$\text{singlets} : \eta', f_0/\sigma, \phi, \ldots$$



Hadronic naming scheme

- dark fermions/quarks: u and d
- mesons named after equivalent QCD state: e.g. dark pions:

$$\pi^+ = \bar{d}\gamma_5 u \quad \pi^- = \bar{u}\gamma_5 d$$

$$\pi^0 = \bar{d}\gamma_5 d - \bar{u}\gamma_5 u$$

$$\eta' = \bar{d}\gamma_5 d + \bar{u}\gamma_5 u$$

The minimal Sp(4) SIMP model

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Sp(4)} + \mathcal{L}_{\mathrm{mediator}}$$

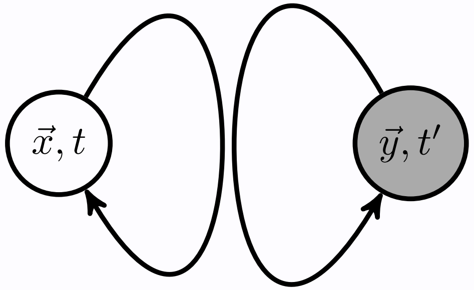
- $Sp(4)$ with $N_f = 2$ has exactly 5 Goldstones
- Dark hadrons DM candidates $\rightarrow$ non-perturbative
- Low energy effective theory (EFT) needed

- **Combine the methods with lattice field theory**
  - Derive low energy EFT for dark sector + mediator
  - Low energy constants (LECs) from lattice
  - Use EFT for astro/collider/direct detection pheno

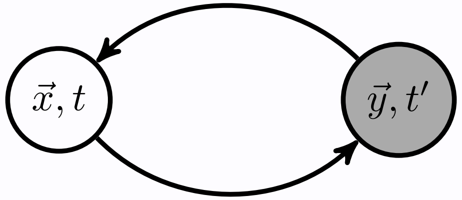
Calculating the meson correlator

- Evaluate diagrams in terms of fermion propagator D^{-1}

$$C(t-t') = \sum_{\vec{x},\vec{y}} \Bigg($$



$$+$$

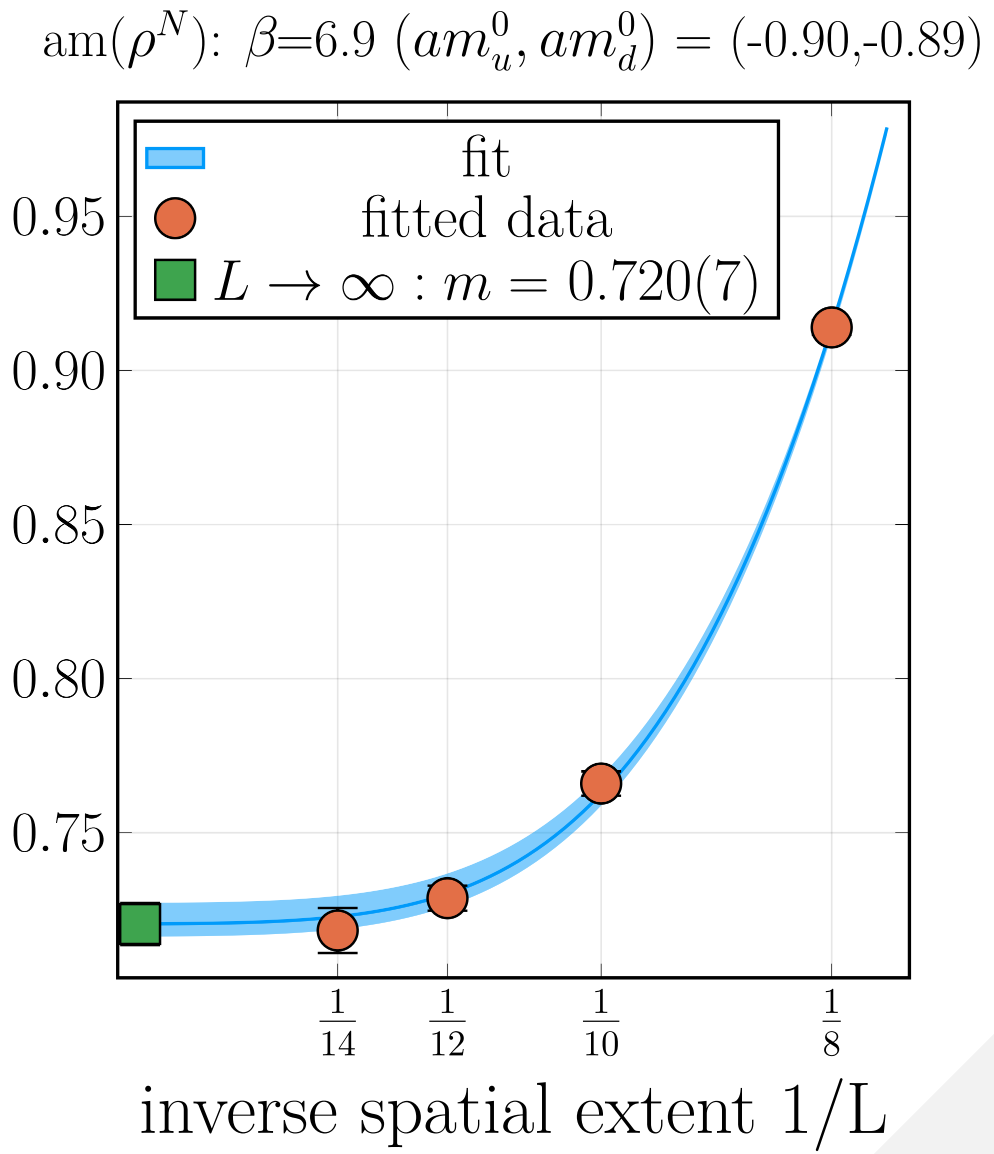


$$\Bigg) + \underbrace{\text{const.}}_{=|\langle 0|O|0\rangle|^2}$$

- Disconnected diagram (left) particularly challenging
  - only appears for singlets (gluonic propagation)
- Constant term arises for singlets
  - vacuum term for σ, fixed topological charge for η'

Lattice technicalities

- Inversion of D: huge matrix typically $N \times N$ where $N \approx \mathcal{O}(10^{12})$
- Disconnected diagrams
- Constant contribution of $|\langle 0|O|0\rangle|^2$
- Infinite volume extrapolation $L \to \infty$
- Finite spacing analysis $a \to 0$



A note on sampling the path integral

- Fermionic action is Grassmann-valued

$$\langle O \rangle = \frac{1}{Z_g Z_f} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] O[A_\mu, \psi, \bar{\psi}] \exp\left(-S_\mathrm{g}[A_\mu] - S_\mathrm{f}[A_\mu, \psi, \bar{\psi}]\right)$$
$$= \frac{1}{Z_g} \int \mathcal{D}[A_\mu] O_F[A_\mu] \det\left(-D[A_\mu]\right) \exp\left(-S_\mathrm{g}[A_\mu]\right)$$

- Lattices are huge: Monte-Carlo sampling required
- Interpret the exponential term and determinant as probability

$$\rho[A_\mu] = \frac{1}{Z_g} \underbrace{\det\left(-D[A_\mu]\right)}_{\text{causes problems}} \exp\left(-S_\mathrm{g}[A_\mu]\right)$$

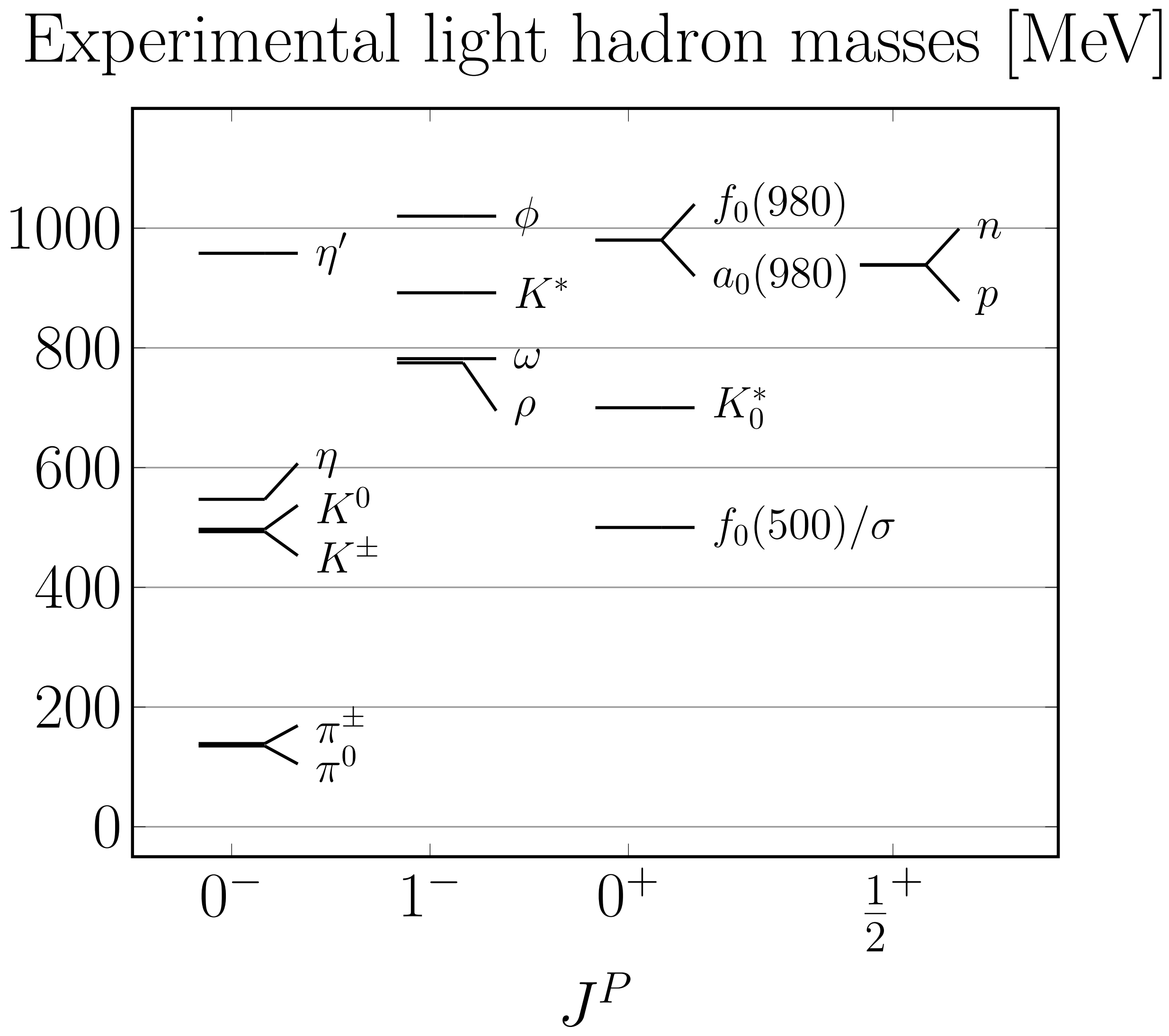
The fermion determinant $\det\left(-D[A_\mu]\right)$

- Not guranteed to be positive definite: Weak sign problem
- But its square is: $\det D \det D = \det\left(DD^\dagger\right) \geq 0$

  two-flavour theories are particularly nice on the lattice!
- Fermions sampled from auxiliary bosonic fields

$$\det\left(DD^\dagger\right) = \pi^{-N} \int_{\mathbb{R}^{2N}} \mathcal{D}\left[\phi, \phi^\dagger\right] \exp\left(-\phi^\dagger \left(DD^\dagger\right)^{-1} \phi\right)$$

```
We can now calculate arbitrary operator
expectation values on the lattice!
```



QCD like Dark Matter models

- Strongly-interacting, confining sector
- Mostly models with dark chiral symmetry breaking
  - pseudo-Goldstones π as DM candidate
- Pheno calculations rely on effective field theories (EFTs)

Composite Higgs studies can be repurposed

- Coset spaces for Higgs physics are large enough for SIMP DM
  - applies also to different fermion reps. (e.g. $Sp(4), N_f^{as} \geq 2$)
  - or mixed representation theories
- Particle spectrum determines relevant hadronic states
- Scattering studies in the context of WW scattering

[1]Nogradi,Szikszai[2107.05996][2]Bennett et.al.[2010.15781][3]Bennett et.al.[2202.05516],Drach et.al.[2107.09974]
[4] e.g. Boz. et.al. [1912.10975], [5] Mason et.al. [2310.02145]

Applications of $Sp(2N)$ gauge theory beyond SIMP DM

- Generic features of non-Abelian confining gauge thoeries
  - Hadron masses as functions of N_f and N_c [1]
  - large N_c limit [2]
- Higgs compositeness, partial top compositeness [3]
  - Mixed fermion representations: near conformal behaviour?
- Model theory for finite density calculations (no sign problem) [4]
- Finite temperature behaviour: Deconfinemt and chiral symmetry
  - Potential first oder phase transitions? [5]

Extra meson states:

Diquarks and Anti-Diquarks

$$\pi_1 : \quad \bar{u}\gamma_5 d$$

$$\pi_2 : \quad \bar{d}\gamma_5 u$$

$$\pi_3 : \quad \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right)$$

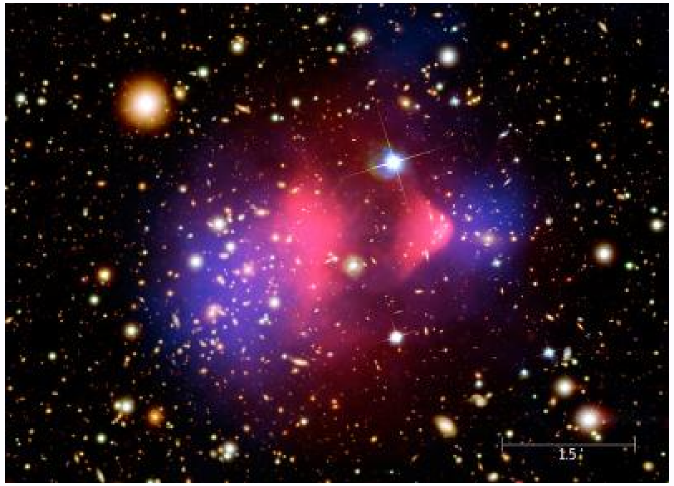
$$\pi_4 : \quad \bar{d}\gamma_5 SC\bar{u}^T$$

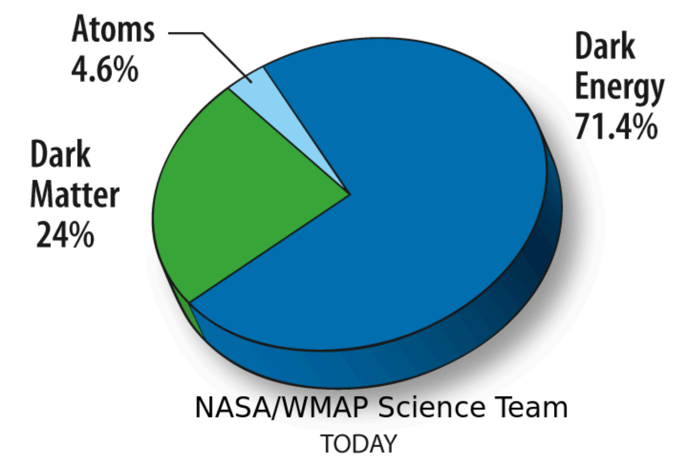
$$\pi_5 : \quad d^T SC\gamma_5 u$$

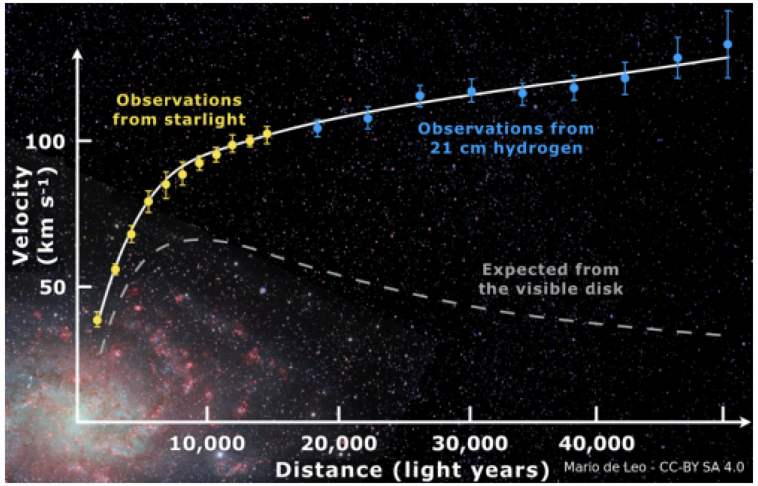
[1] e.g. Bertone, Hooper, Silk. [hep-ph/0404175] [2] e.g. PDG review and Famaey, McGaugh [1112.3960]

Dark Matter - Why?

- Strong observational evidence at many scales!
- Modified gravity, primordial black holes are alternatives
- New particles beyond the Standard Model (BSM) promising!



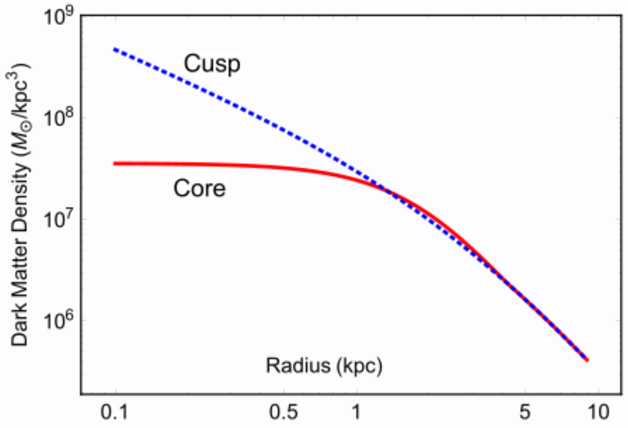


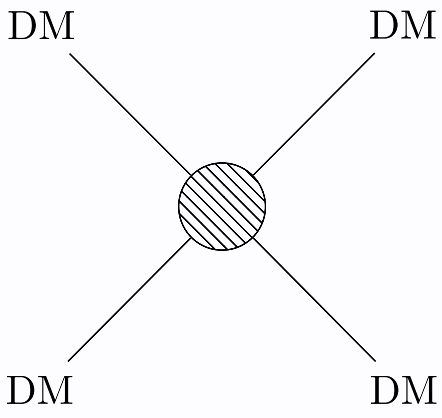


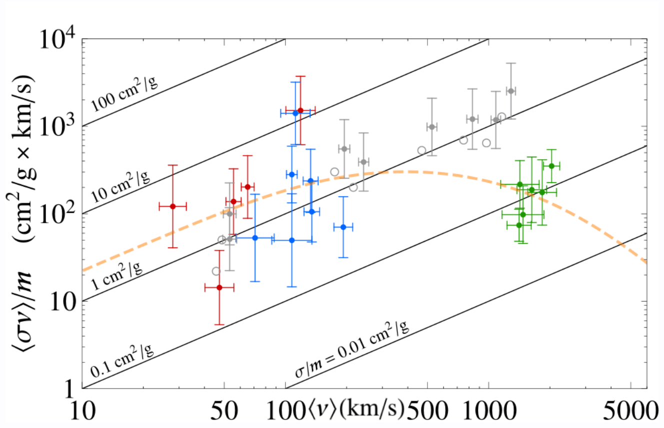
[1] see e.g. Bullock,Boylan-Kolchin [1707.04256], Tulin, Yu [1705.02358]

Dark Matter properties

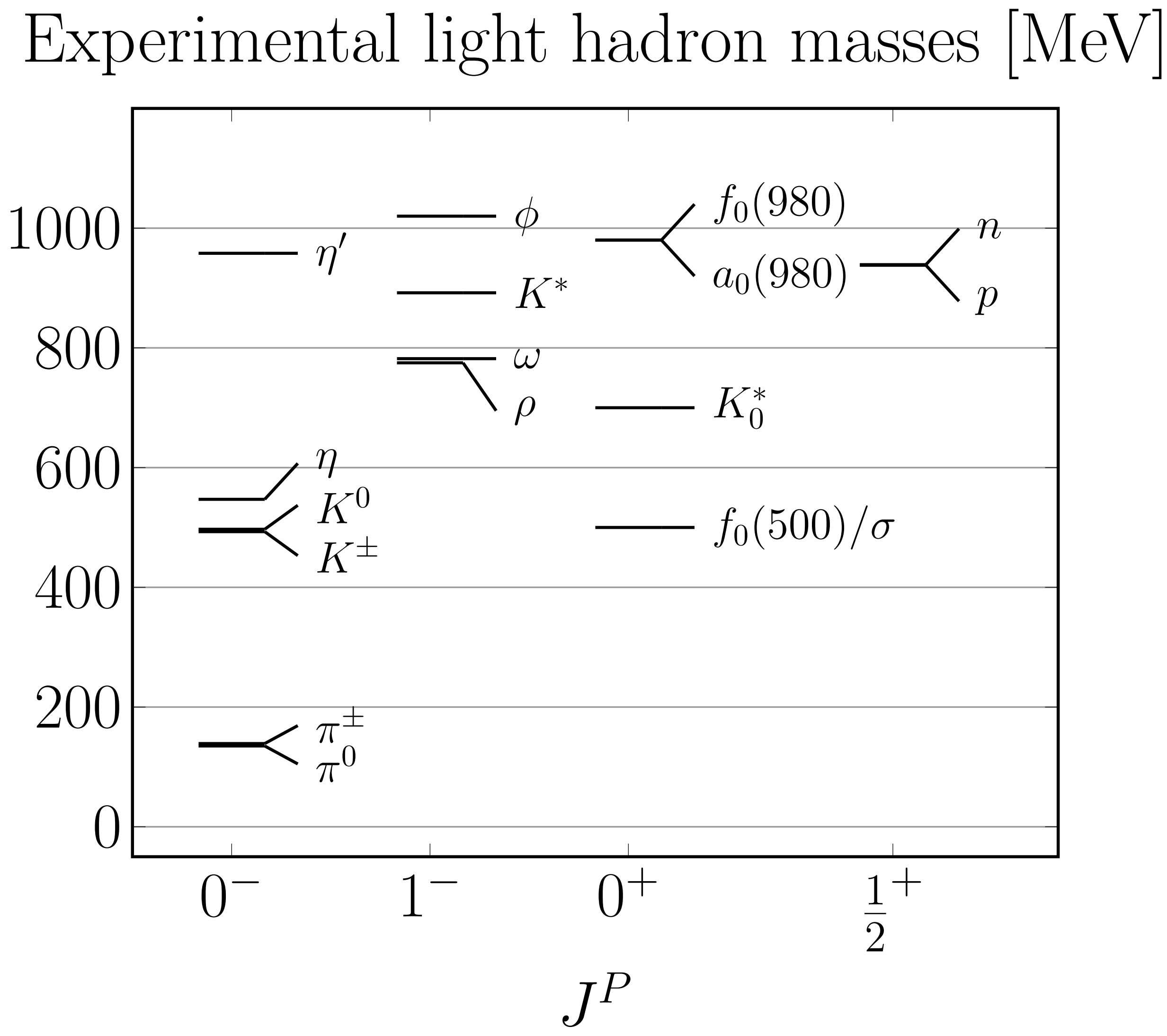
- DM self-interaction phenomenologically allowed[1] and potentially relevant for small-scale structure problems
  - non-vanishing scattering cross-sections $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$
  - velocity dependence of $\sigma_{2\mathrm{DM}\to 2\mathrm{DM}}$ preferred







QCD-like Dark Matter can those provide self-interactions!



QCD Spectrum

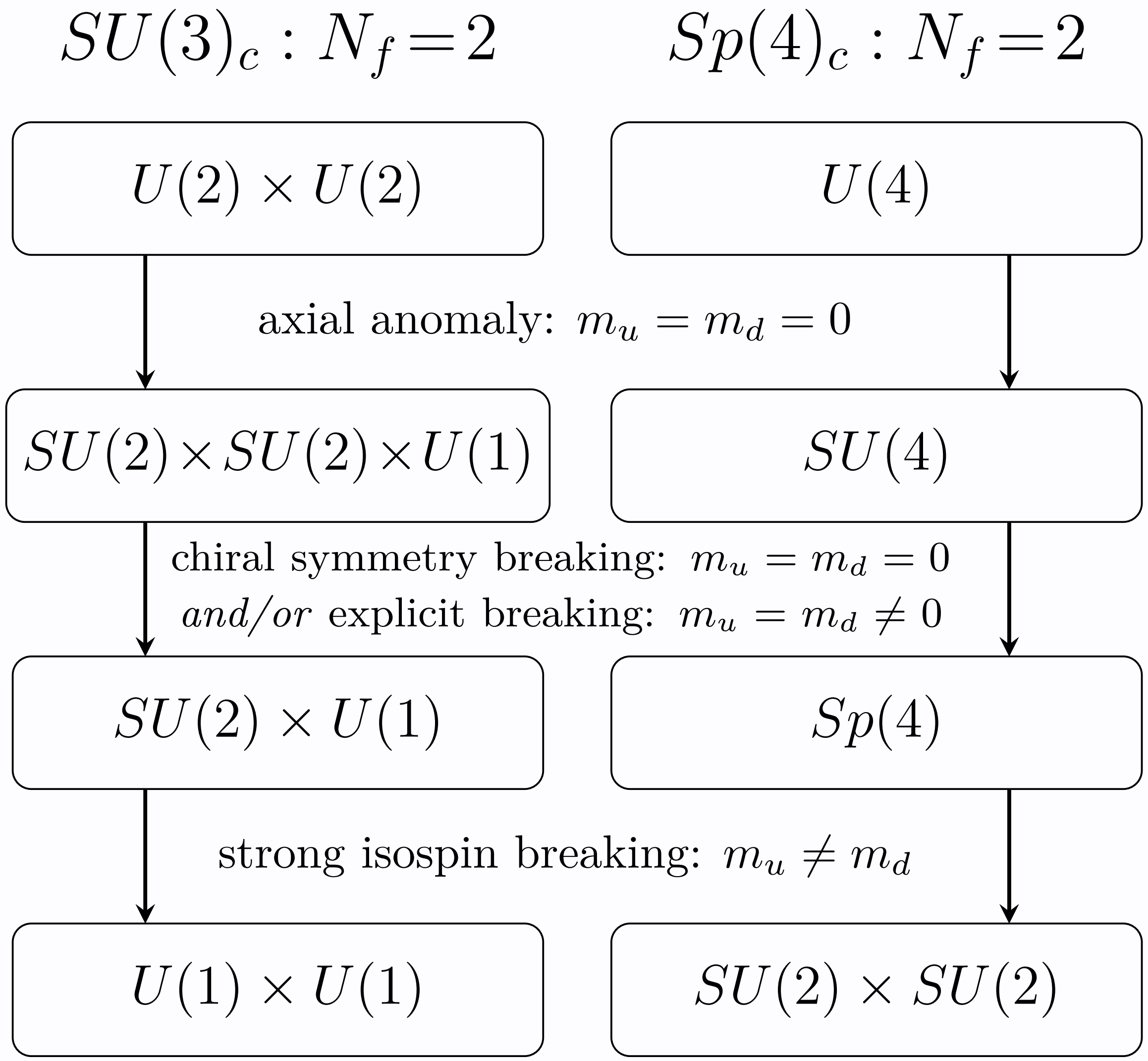
- π, K, η light: pseudo-Goldstones
- Vectors and scalars light
- Light and broad 0^+ singlet f_0/σ
- Heavy 0^- singlet η'
  $\Rightarrow U(1)_A$ anomalously broken

A concrete model theory:

Two-flavour Sp(4) Gauge Theory

[1] Kosower (Phys.Lett.B. 1984)

[2] Hochberg et. al. [1411.3727] [1512.07917]



SIMPs from $Sp(4)$ gauge theory

- Pseudo-real representation: [1]
  $\Rightarrow$ more pseudo-Goldstones
  $\Rightarrow$ no fermionic bound states
- $N_f = 2$: exactly 5 Goldstones
  - Allows $3\mathrm{DM} \rightarrow 2\mathrm{DM}$ [2]

```
Sp(4) with two fermions is
a minimal SIMP DM realisation
```

Lagrangian of $Sp(4)_c$ with fermions

$$\mathcal{L}_{Sp(4)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f(i\not{D} + m_f)\psi_f$$

- Higher symmetry than QCD-like theories

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

$C \ldots$ charge conj.
$S \ldots$ colour matrix

$$\mathcal{L}_{Sp(4)} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- generators τ_a in fundamental repr. : $S\tau_a S = -\tau_a^T$
- mass matrix M proportional to symplectic invariant

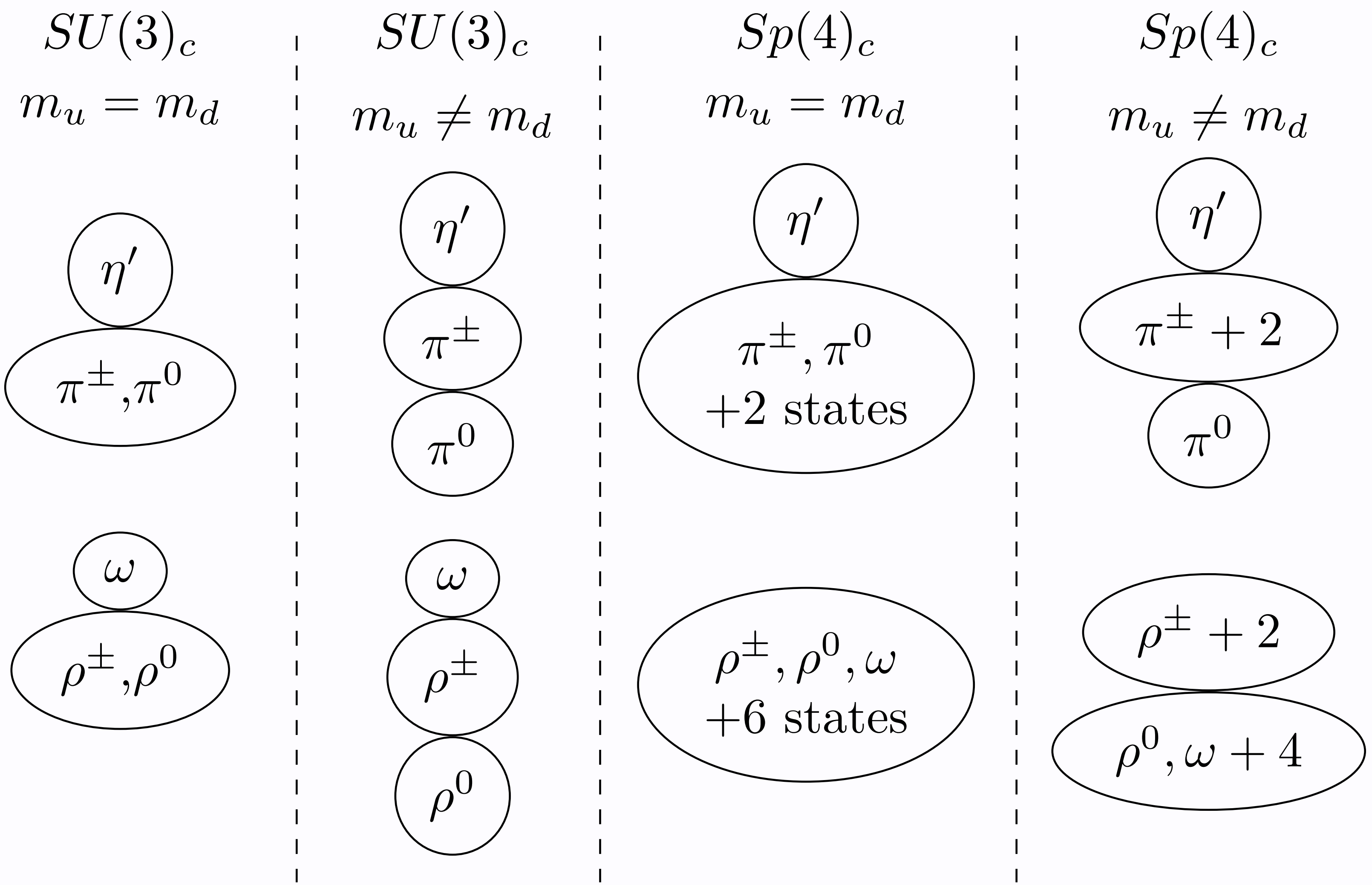
Meson multiplets of $Sp(4)_c$ with $N_f = 2$

- $Sp(2N_f)$ flavour symmetry between $2N_f$ Weyl components
- Extra gauge invariant states: $q^T \dots q$ and $\bar{q} \dots \bar{q}^T$

$$Sp(4)_F : \qquad 4 \otimes 4 = 1 \oplus 5 \oplus 10$$

```
The global symmetries lead to a richer meson multiplet structure!
```

Pseudoscalar (PS) and vector (V) multiplets



The same patterns persist for other channels.

Non-perturbative input is needed:

The case for lattice investigations

The case for lattice investigations

- Theory is non-perturbative at low energies!
  - Lattice allows first-principles calculations
  - Errors are systematically improveable
- Effective field theories are powerful tools!
  - Lattice can calculate low-energy constants
  - provides connection to UV complete theory
- Scattering properties accessible on the lattice!

BSM/DM wishlist from the lattice

1. Masses and decay constants of dark hadrons
   - Non-singlet and singlet mesons, glueballs
2. Scattering of dark pions
   - $2\pi \to 2\pi$ for self-interaction crossection
   - $3\pi \to 2\pi$ for SIMP semi-annihilation
3. Applicability of χPT and related EFTs

Lattice spectroscopy: Getting meson masses

- Construct operator with same quantum numbers, e.g.

$$O_\pi = \bar{u}\gamma_5 d \qquad\qquad J^P = 0^-(\textbf{non-singlet})$$

- Spectroscopy for meson from its correlator $C_M(t)$

$$C(t = \tau - t') = \sum_{\vec{x},\vec{y}} \langle O(\vec{x},\tau) O^\dagger(\vec{y},t') \rangle$$

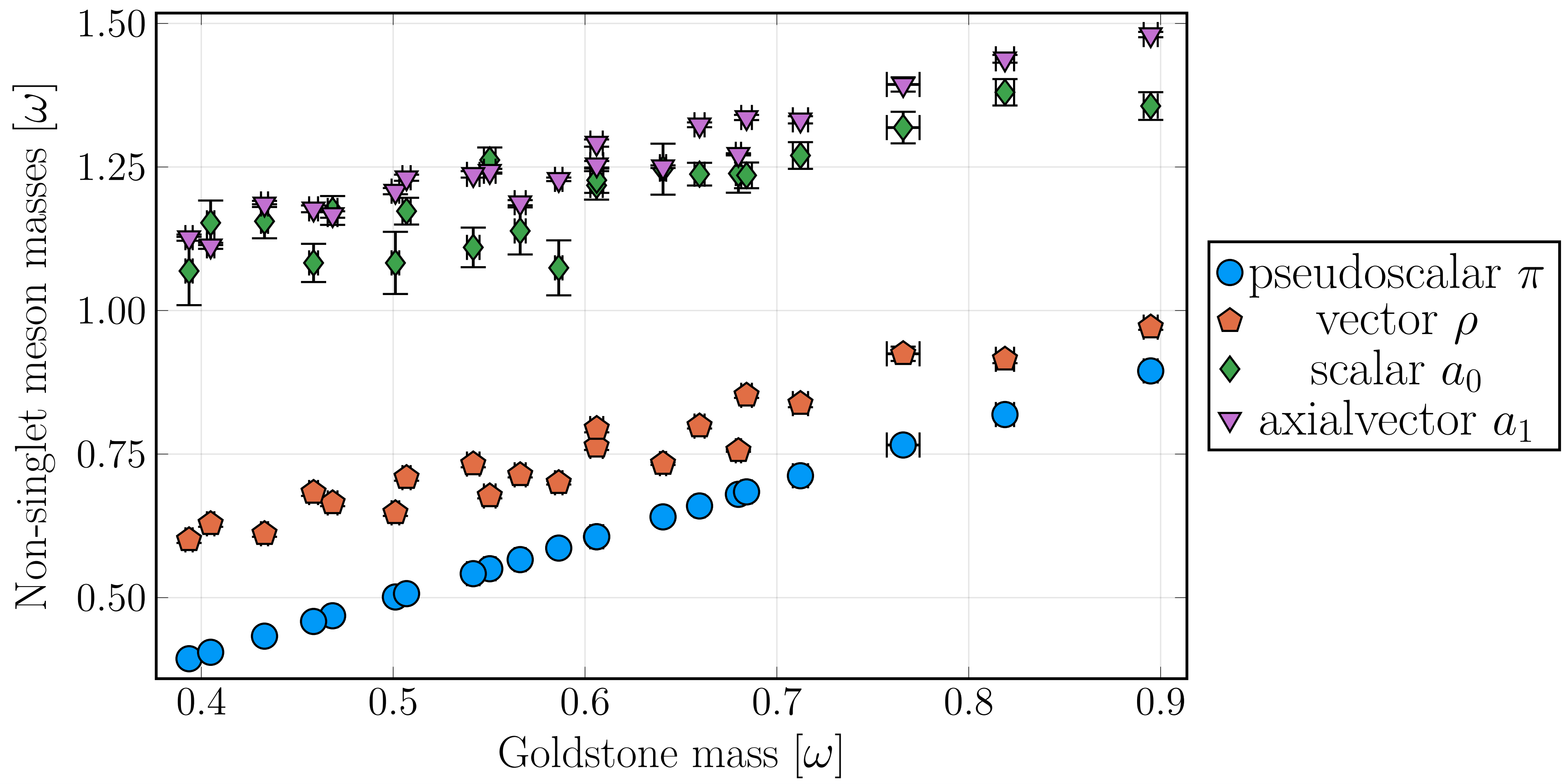
$$= \sum_{\vec{x},\vec{y},n} \langle 0 | O(\vec{x},\tau) | n \rangle \langle n | O^\dagger(\vec{y},t') 0 | \rangle \frac{e^{-E_n t}}{2E_n}$$

$$C(t = \tau - t') = A e^{-E_0 t} + \mathcal{O}(e^{-\Delta E t})$$

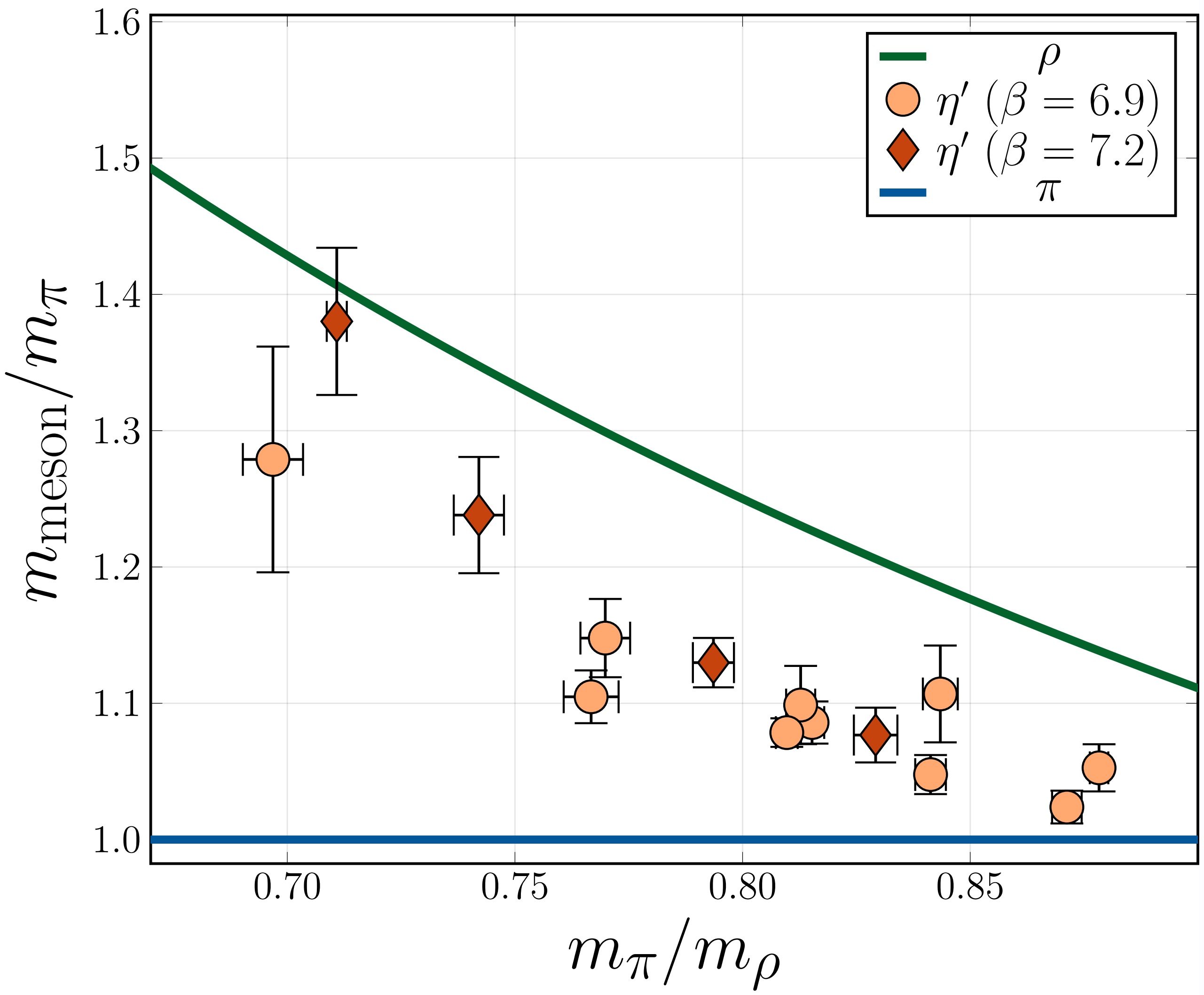
- Ground state mass $E_0 = m$, decay constant $\propto \sqrt{A}$

Bennett et. al. [1909.12662]

Non-singlet spectrum



The pseudoscalar and vector mesons are the lightest non-singlets.



The pseudoscalar singlet η' is surprisingly light!

- Phenomenologically relevant:
  - $m_\rho > m_{\eta'}$ different from QCD
  - relevant low-energy dof
  - η' relevant for $\pi\pi$ scattering
  - more accessible channels for decays into SM

Interesting! Is this surprising?

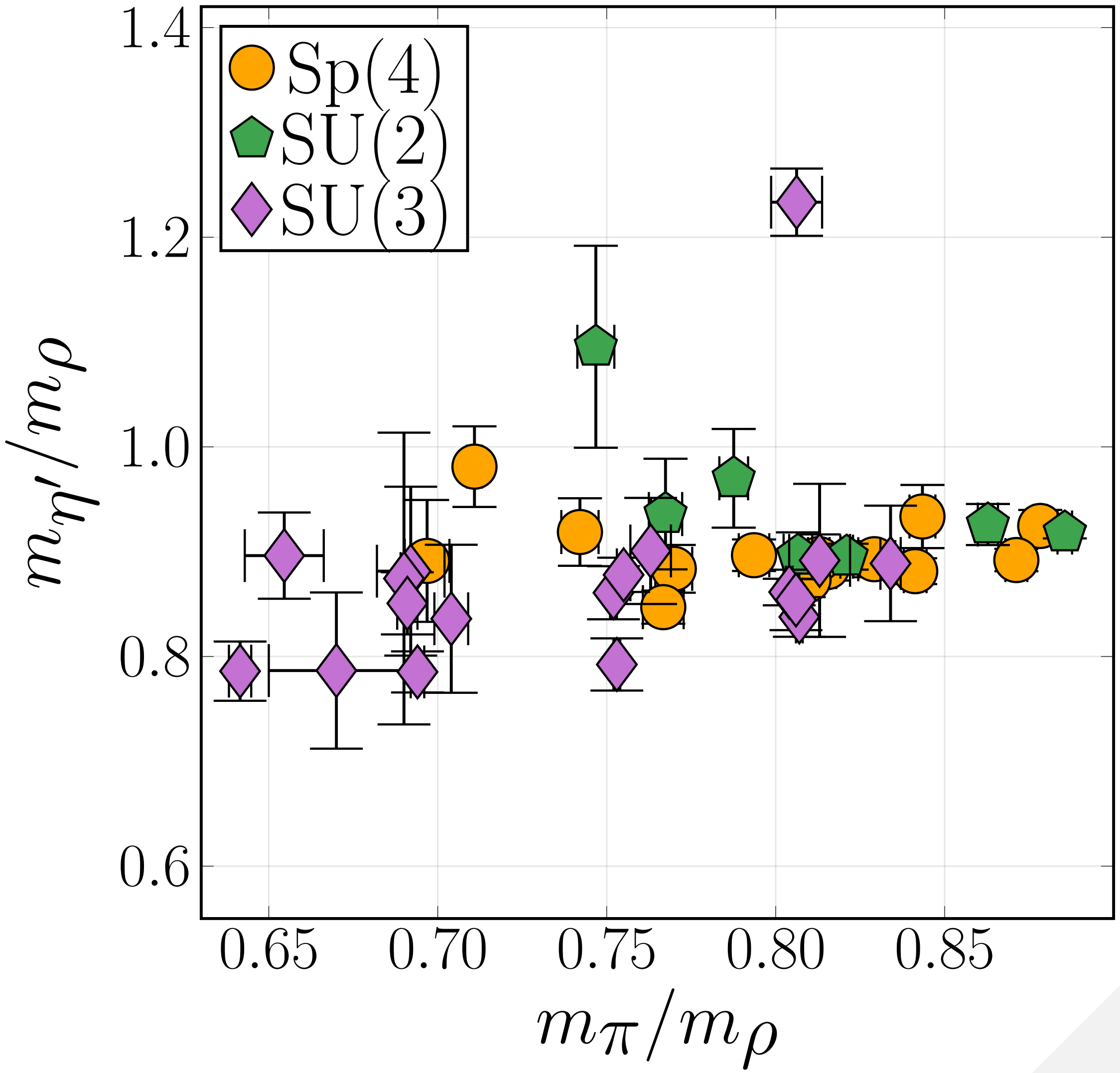
Consider different theories:

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"

SU(2) and SU(3) comparison:

- Similarities:generic $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi / m_\rho \to 0$

mass driven by flavour content!



Consequences for Dark Matter

- Mass hierarchies: limit χPT validity
  - inclusion of other states than π required, e.g. η' and ρ
  - additional tests needed (fermions are too heavy)
- Light unprotected states η', π^0 allow decays into SM
  - no protection from symmetry

```
Are these fermion masses phenomenologically relevant?
```

Dark Matter Scattering on the Lattice

- Pions are in the 5-dimensional representations
- A two pion scattering is in one of three irreps

$$\mathbf{5} \times \mathbf{5} = \mathbf{14} \oplus \mathbf{10} \oplus \mathbf{1}$$

- Corresponds to the usual QCD channels
  - $\mathbf{14} \Leftrightarrow$ isospin $I = 2$ in QCD, e.g. $\pi^+\pi^+$
  - $\mathbf{10} \Leftrightarrow$ isospin $I = 1$ in QCD, e.g. $\pi\pi \to \rho$
  - $\mathbf{0} \Leftrightarrow$ isospin $I = 0$ in QCD, e.g. $\pi\pi \to \sigma/f_0$

[1] M. Lüscher (Nucl.Phys.B 1991)

Scattering information from the lattice

- Scattering phase shift $\delta_0(p)$ from finite volume energy

$$\tan(\delta_0(q)) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \qquad q = p^* \frac{L}{2\pi}$$

$$\cosh\left(\frac{E_{\pi\pi}}{2}\right) = \cosh(m_{\pi\pi}) + 2\sin\left(\frac{p^*}{2}\right)^2$$

- Low-velocity behaviour: Scattering length

$\Rightarrow$ relation between $\pi\pi$ energy $E_{\pi\pi}$ and m_π on a lattice [1]

$$\boxed{\frac{\delta E_{\pi\pi}}{m_\pi} = \frac{4\pi m_\pi a_0}{(m_\pi L)^3}\left(1 + c_1 \frac{m_\pi a_0}{m_\pi L} + c_2 \left(\frac{m_\pi a_0}{m_\pi L}\right)^2\right)}$$