Beyond the Standard Model Spectroscopy with Lattice Gauge Theory

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(with the TELOS collaboration)

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Overview

1. Strongly Interacting BSM models
    - Strongly Interacting Dark Matter
    - Composite Higgs Models
    - QCD-like Models and Global Symmetries
2. Lattice QCD & Lattice Gauge Theory
3. Lattice Results for BSM models
    - Spectroscopy
    - (Scattering)
    - (Finite temperature behaviour)

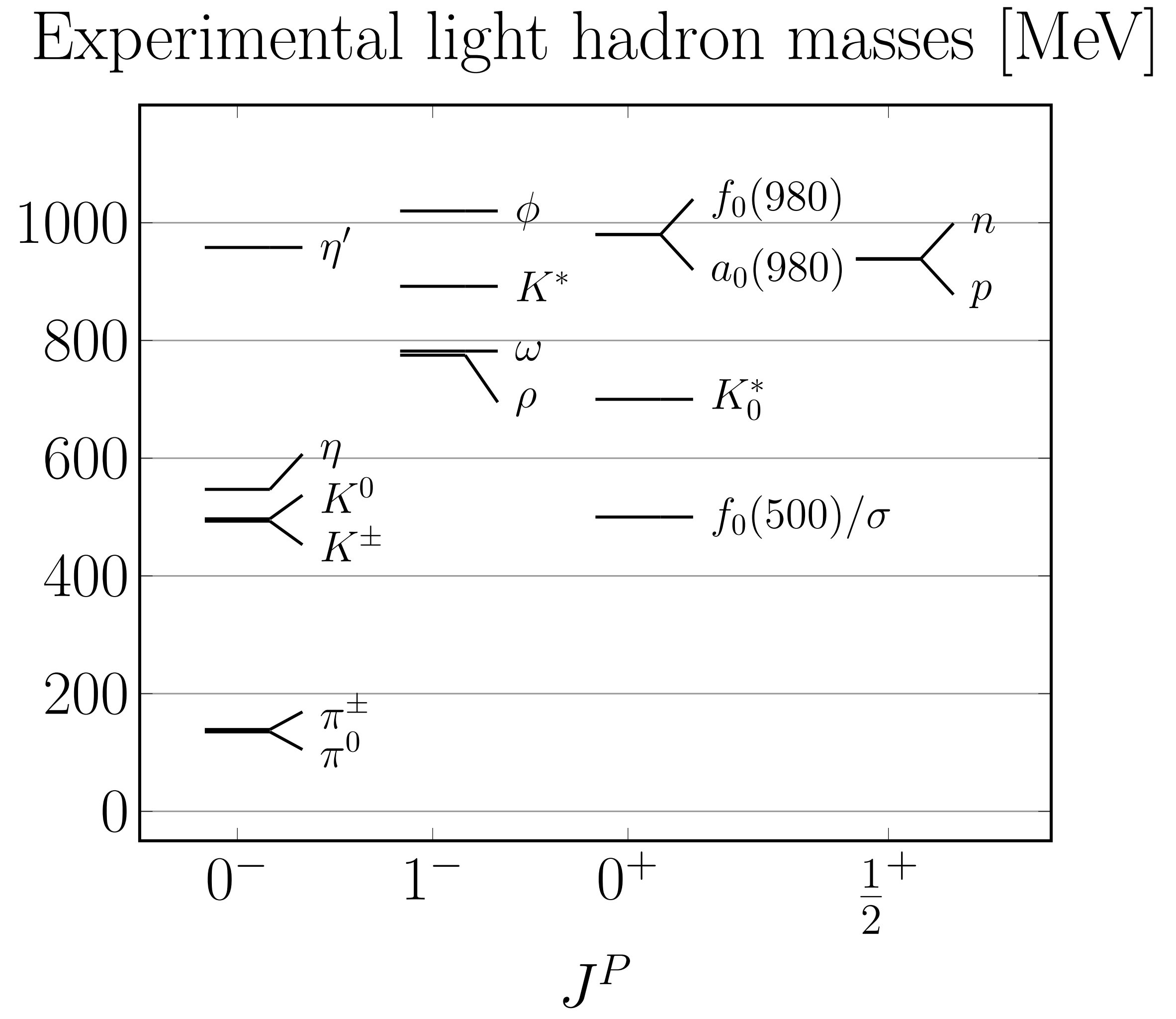
1) Strongly Interacting BSM models

The Standard Model is not complete

- Many other SM extensions exists
- Among them some are **QCD-like** in some sense
- Studied in multiple variations
  - pure gauge: Deconfinement transition, glueballs, ...
  - with Fermions: **composite Higgs**,**composite Dark Matter**
- UV complete due to asymptotic freedom

- This talk: Class of QCD-like BSM models

Spectrum and Symmetries in QCD



- Set of parametrically light states: π
- Approximate flavour symmetries $SU(2)_F$ from $m_u \approx m_d$
- Lagrangian has larger symmetry for $m_u = m_d = 0$ $\Rightarrow$ spontaneously broken $\Rightarrow$ π:**pseudo-Goldstone**

Structure of Goldstone BSM extensions

- **Inspired by the symmetries and spectrum of QCD**
- Postulate an additional confining BSM sector
- Spectrum consists of bound states

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{BSM}}$$

$$\mathcal{L}_{\mathrm{BSM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f \left(i \not{D} + m_f \right) \psi_f + \left(L_{\mathrm{mediator}} \right)$$

- Contains **parametrically light** bound states:
- *pseudo-Goldstone boson of global symmetry breaking*

see e.g. Kogut et. al. [hep-ph/0001171], von Smekal [1205.4205]

Global Symmetries of QCD-like Theories

- Global symmetry determined by fermion irrep
  1. complex (e.g. QCD, fundamental $SU(N_c \geq 3), \ldots$)
  2. pseudo-real (e.g. fundamental $SU(2)$, $Sp(N_c = 2N), \ldots$)
  3. real (e.g. adjoint reps, $SO(N_c), \ldots$)
- (Pseudo-)Real: Different chiral symmetry breaking pattern
- (Pseudo-)Real: Larger hadronic multiplets

  # of Goldstones: **Dim. of** G/H for breaking pattern $G \rightarrow H$

see e.g. Kogut et. al. [hep-ph/0001171], von Smekal [1205.4205]

Unitary and Symplectic Gauge Groups

$$\mathcal{L}_{\mathrm{BSM}} = \bar{\psi}\gamma_\mu \underbrace{(\partial_\mu + iA_\mu)}_{D_\mu}\psi + \bar{\psi}M\psi$$

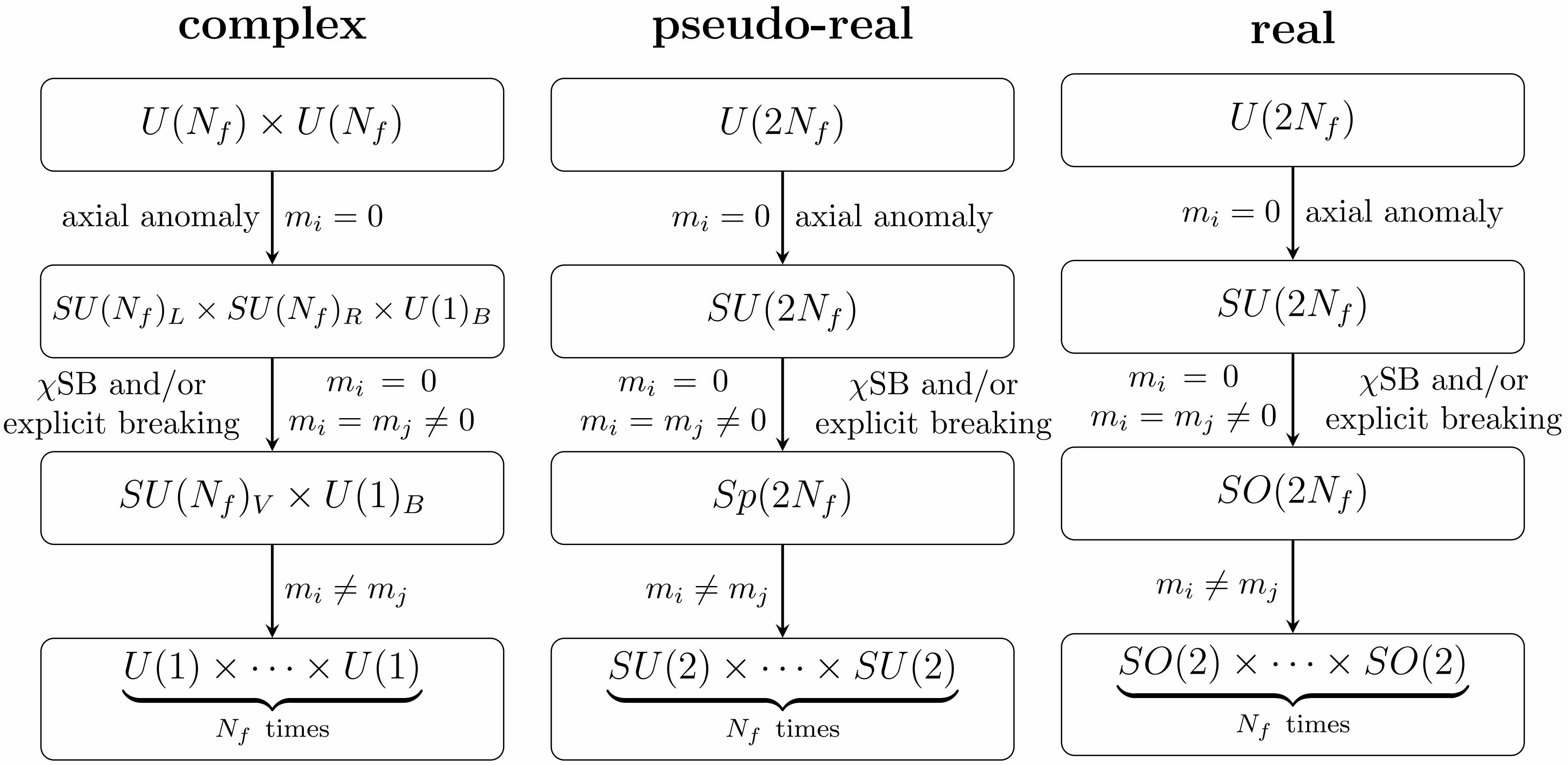
- A_μ from some gauge group G, ψ in an irrep of G
- Complex: Kinetic terms does not mix left & right
- (Pseudo-) Real: Symmetry between Weyl components

$$\Psi = \begin{pmatrix} \psi_L \\ -SC\psi_R^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} \qquad \begin{matrix} C \ldots \text{charge conj.} \\ S \ldots \text{colour matrix} \end{matrix}$$

$$\mathcal{L}_{\mathrm{BSM}} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right) \;\text{ with } S\tau_a S = -\tau_a^T$$

see e.g. Kogut et. al. [hep-ph/0001171], von Smekal [1205.4205], Kulkarni al. [2202.05191]

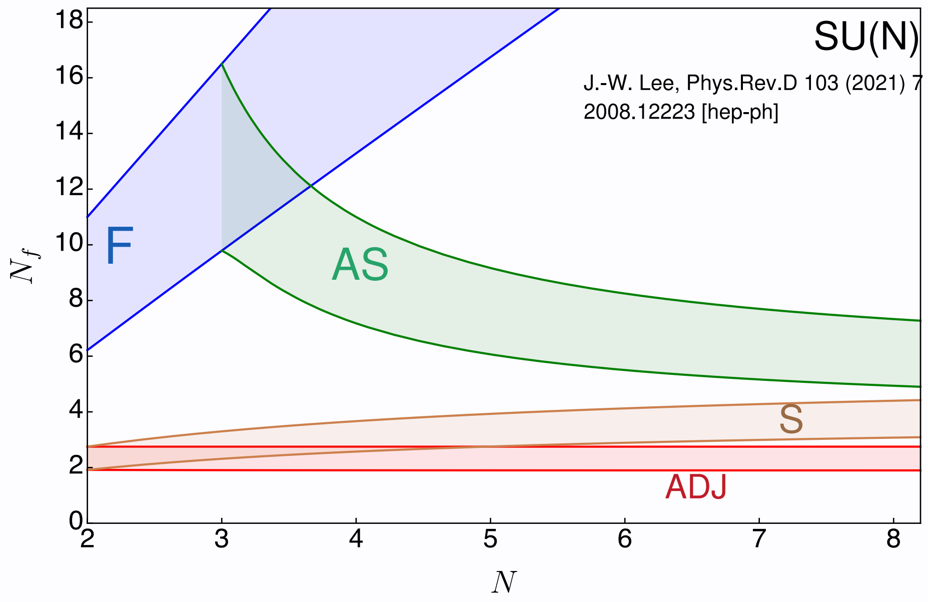
Global Symmetries of QCD-like Theories

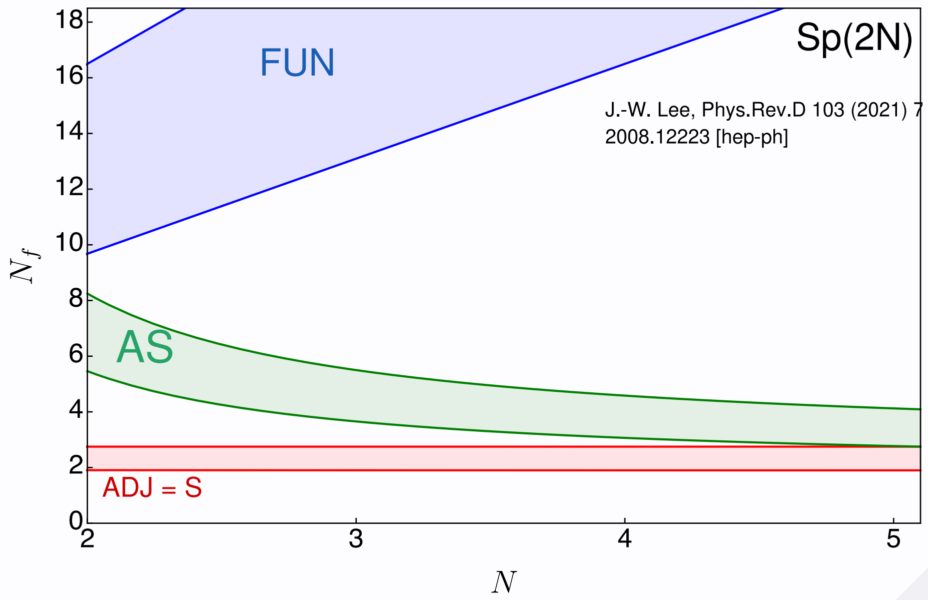


Larger global symmetries useful in model building!

Colour and Flavour Content

- Limit on fermion content
  - asymptotic freedom
  - conformal window
- A priori free to choose:
  - gauge group & N_c
  - fermion irrep & N_f
  - fermion masses m_f
  - gauge coupling $\Leftrightarrow \Lambda_{\mathrm{BSM}}$

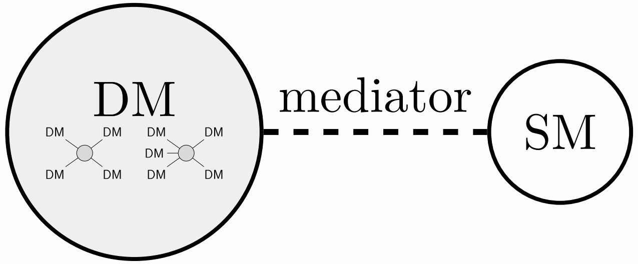




[1] see e.g. Bullock,Boylan-Kolchin [1707.04256], Tulin, Yu [1705.02358]

Strongly Interacting Gauge Theories in DM Models

- With fermions: Global symmetries make DM stable
- With mediator: Dark sector coupled to SM



$$\mathcal{L}_{\rm DM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_f(i\not{D} + m_f)\psi_f$$

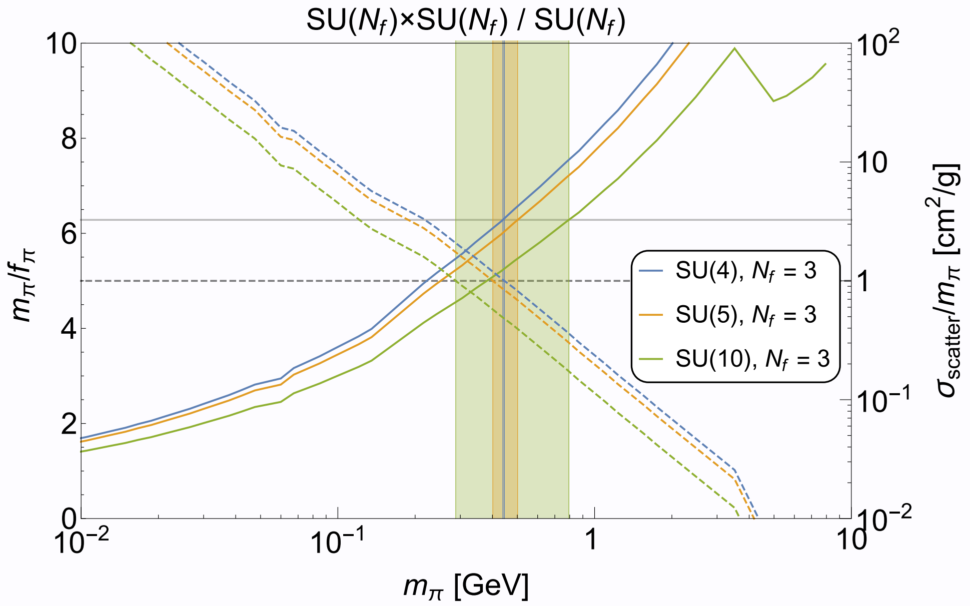
- Dark pions as main Dark Matter candidate
- Non-vanishing self-scattering cross-section arise

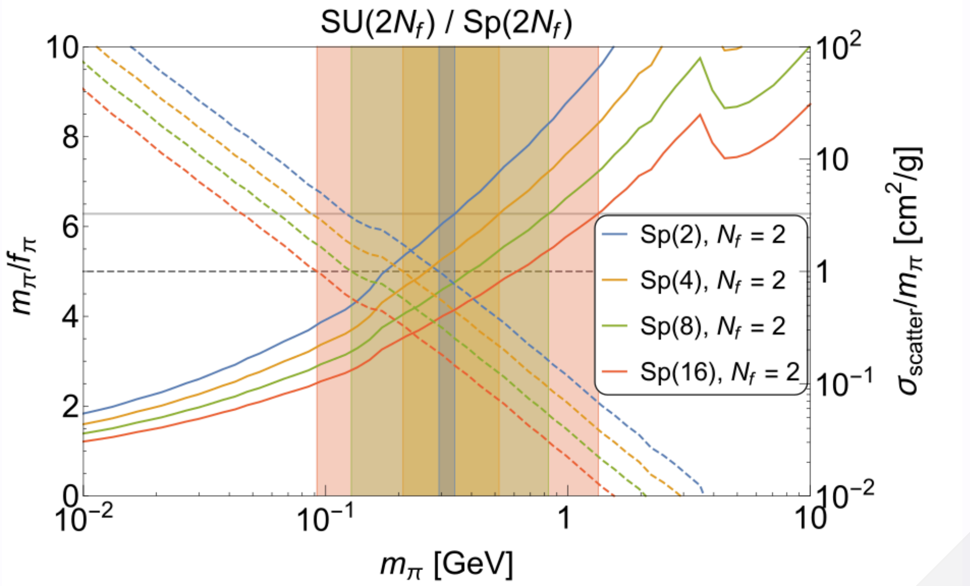
$$\langle v\sigma_{\pi_D\pi_D\to\pi_D\pi_D}\rangle \neq 0$$

- Dark Matter relic density driven by strong processes
- BSM fermion are SM singlets!

Specific Models: Dark Matter

- Example among many:
- **S**trongly **I**nteracting **M**assive **P**articles [1]
  - DM relic density from $3\pi_D \to 2\pi_D$ decay
  - in χPT: Wess-Zumino-Witten term[2]
- Almost any coset possible, pseudoreal preferred





Composite Higgs (+ Composite Top): Constraints

- CHM: Higgs boson is composite particle of BSM fermions
- *Partial top compositeness*: top quark composite
- BSM fermions carry SM quantum numbers $\Rightarrow$ few options
  - need to match *all* quantum numbers of Higgs & top in $G_{\text{custodial}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$
  - (typically) **requires two distinct** fermion irreps

table taken from Feretti [1604.06467], see also Ferretti,Karateev [1312.5330]

[1] Belyaev et.al.[1610.06591], Cacciapaglia et.al.[1902.06890] Franzosi et.al.[2106.12615] DeGrand et.al. [1605.07738]

Composite Higgs + Top Realizations: Possibilities

G_{HC}	ψ	χ	Restrictions	G/H
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\mathrm{HC}} = 7, 9$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 7, 9$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)} \mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)}{\mathrm{SU}(3)_D} \mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$N_{\mathrm{HC}} = 10$	
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\mathrm{HC}} = 4$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)$
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 11$	
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 10$	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\mathrm{HC}} = 4$	
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$N_{\mathrm{HC}} = 5, 6$	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)}{\mathrm{SU}(3)_D} \mathrm{U}(1)$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

- Extra Goldstone boson appears due to additional $U(1)$[1]!

Composite Higgs + Top Realizations: Specific Theory

- $Sp(4)_C$ with $N_f = 2$ fundamental and $n_f = 3$ antisymmetric

$$\mathcal{L}_{\mathrm{BSM}} = -\frac{1}{2}\mathrm{Tr}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \sum_{\mathrm{i}=1}^{2} \bar{\psi}^{\mathrm{i}} \left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{i}}^{\mathrm{f}}\right) \psi^{\mathrm{i}} + \sum_{\mathrm{j}=1}^{3} \bar{\chi}^{\mathrm{j}} \left(\mathrm{i}\not{D} - \mathrm{m}_{\mathrm{j}}^{\mathrm{as}}\right) \chi^{\mathrm{j}}$$

- Symmetry breaking:

$$SU(4)_{\mathrm{F}} \times SU(6)_{\mathrm{AS}} \times U(1) \to Sp(4)_{\mathrm{F}} \times SO(6)_{\mathrm{AS}} \times U(1)$$

- colour-singlet bound states:
  - F mesons $\bar{\psi}\psi$ $(\psi_c\psi)$, AS mesons $\bar{\chi}\chi$, glueballs
  - Mixed-representation baryons ("*chimera baryons*")

  $\Rightarrow$ composite top partner with $QQ\psi$

Effective Field Theories & Non-Perturbative Physics

- Powerful EFT for Goldstones: Chiral Perturbation Theory!
  - extended to arbitrary gauge group G & fermion rep
  - can be extended to include additional states: $\rho, \eta', \ldots$
  - has been extended for multiple irreps
- But **requires non-perturbative** input
  - Low energy constants free parameters of EFT
  - EFT has limited applicable energy range!

Goal: Supplement EFTs and go beyond their applicability with non-perturbative calculations from first-principles

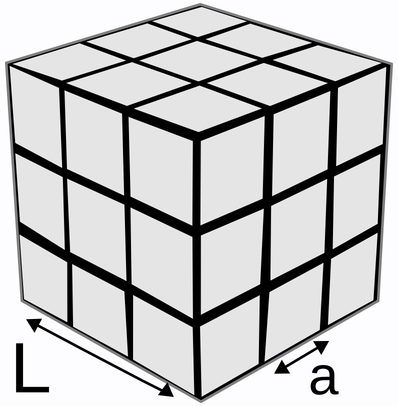
2) Lattice QCD & Lattice Gauge Theory

The Lattice as a Regulator

- *Euclidean* action S on hypercubic lattice

$$Z = \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]}$$

- Discretized spacetime
- lattice spacing a
- volume $V = L^4 = (aN_L)^4$



- Lattice regulator

  UV: finite **spacing** a

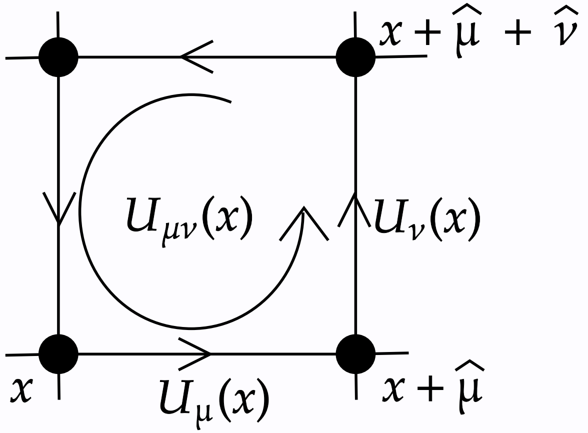
  IR: finite **extent** L
- Continuum: $a \to 0$, $L \to \infty$
- Regulator *allows non-perturbative calculations*

Discretized Lattice Action

- Lattice sites $\Leftrightarrow$ fermions, links U_μ $\Leftrightarrow$ gauge fields
- Lattice action not unique. Here: Standard Wilson action

$$S_g[U_\mu] = \frac{2}{g^2} \sum_{x,\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\,[1 - U_{\mu\nu}(x)] \qquad S_f[U_\mu, \psi, \bar{\psi}] = a^4 \sum_{x,y} \bar{\psi}(x) D(x|y) \psi(y)$$

$$D(x|y) = (m + 4/a)\,\delta_{x,y} - \frac{1}{2a} \sum_{\mu=\pm 1}^{\pm 4} (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu},y}$$



- Recovers continuum action for limit $a \to 0$
- Leading corrections $\mathcal{O}(a)$

The Lattice as a Computational Tool

- Solve $\langle O \rangle$ numerically, with Monte-Carlo methods
- Integrate out fermions analytically

$$\langle O \rangle = \frac{1}{Z_g Z_f} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] O[A_\mu, \psi, \bar{\psi}] \exp^{-S_\mathrm{g}[A_\mu] - S_\mathrm{f}[A_\mu, \psi, \bar{\psi}]}$$

$$= \int \underbrace{\frac{1}{Z_g} \mathcal{D}[A_\mu] \det\left[-D[A_\mu]\right] \exp^{-S_\mathrm{g}[A_\mu]}}_{\text{interpret as probability distribution } P[A_\mu]} O_F[A_\mu]$$

- Sample gauge fields from $P[A_\mu]$ & evaluate $O_F[A_\mu]$

$\Rightarrow$ allows non-perturbative evaluation of $\langle O \rangle$

Spectroscopy on the Lattice

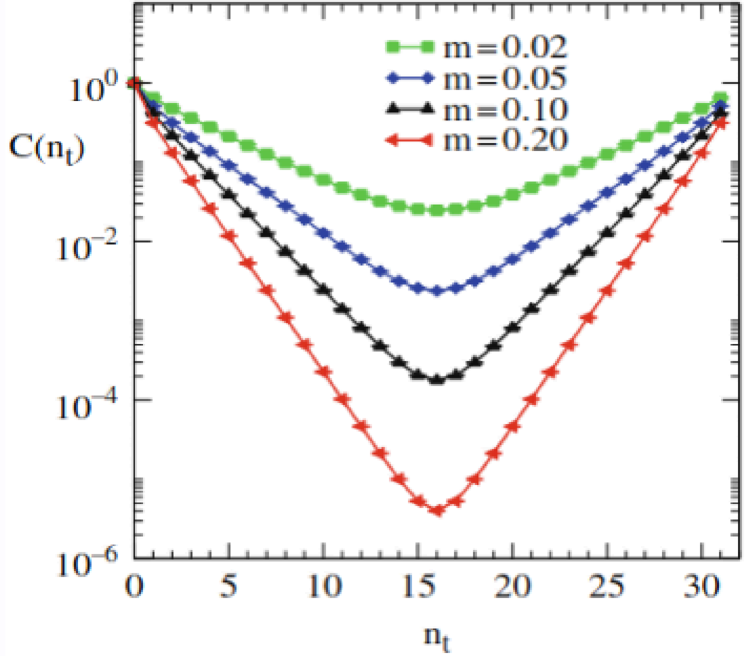
- Operator with desired quantum numbers, e.g. $O_\pi = \bar{u}\gamma_5 d$
- Consider Euclidean correlator $C(t)$

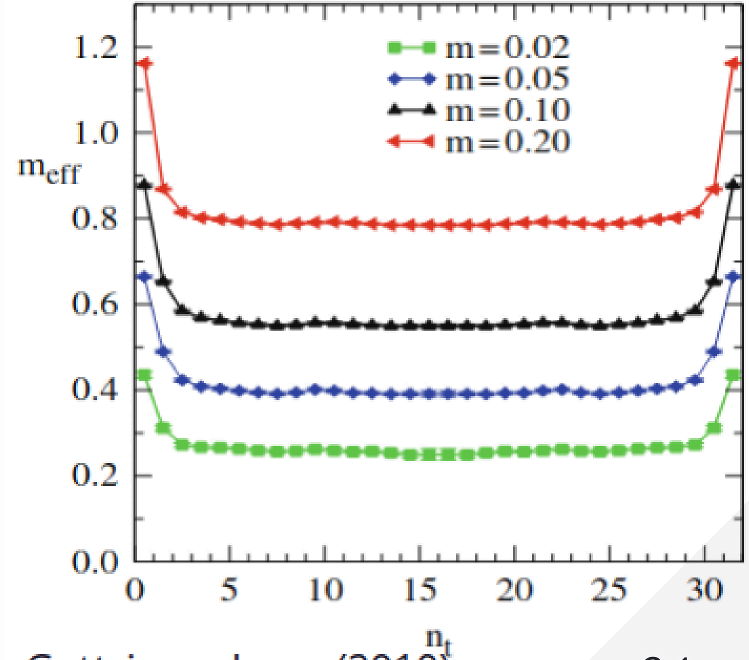
$$C(t=\tau-t') = \sum_{\vec{x},\vec{y}} \langle O(\vec{x},\tau) O^\dagger(\vec{y},t')\rangle$$

$$= \sum_{\vec{x},\vec{y},n} \langle 0|O(\vec{x},\tau)|n\rangle\langle n|O^\dagger(\vec{y},t')0|\rangle \frac{e^{-E_n t}}{2E_n}$$

$$C(t=\tau-t') = Ae^{-E_0 t} + \mathcal{O}(e^{-\Delta E t})$$

- Energy levels E_n accessible!





Gattringer,Lang (2010)
Lect.Notes Phys.788

3) Results from the TELOS collaboration

TELOS Collaboration

Theoretical Explorations on the Lattice with Orthogonal and Symplectic groups.

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Nuno Brito
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University of Plymouth
kzone-dev

Paul Xiao
University of Tsukuba
theHoHsiao

Other investigations by different collaborations with various gauge groups

- Pure Gauge: $SU(N_c \leq 10)$, $Sp(N_c \leq 8)$, G_2, $\ldots$
- Fundamental irrep: $SU(2,3,4)$, $Sp(4)$, G_2, $SO(4)$, G_2, $\ldots$
- Others:$SU(3)$Adj, $SU(2)$Adj, $SU(3)$S, $Sp(4)$AS, $SU(4)$S,$\ldots$
- Mixed: $SU(2)$Adj+F, $SU(4)$S+F, $Sp(4)$AS+F

- In some cases results with various N_f available

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AS: anti-symmetric irrep, S: symmetric irrep, Adj: adjoint irrep
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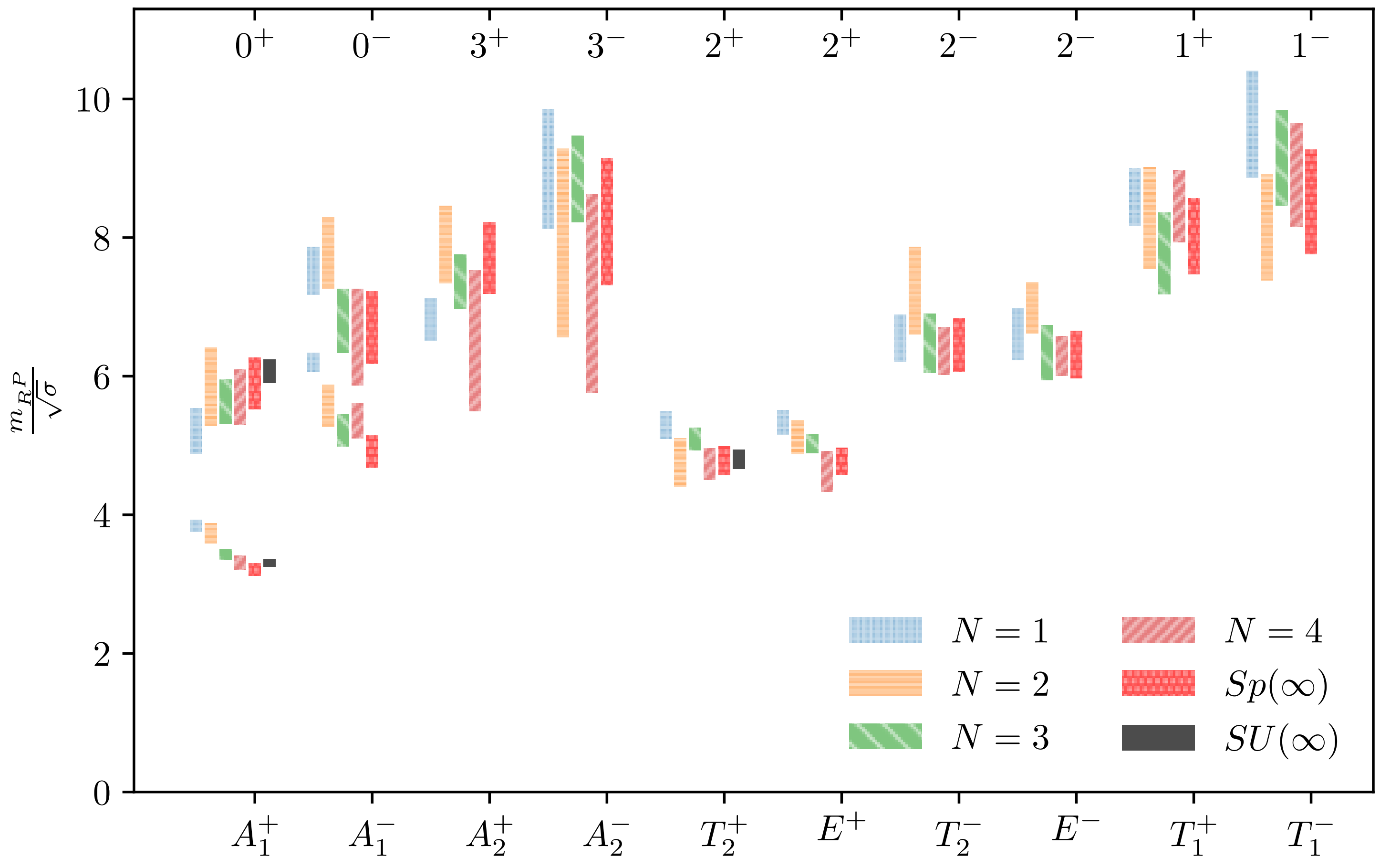
Bennett el.al. (TELOS)[1712.04220] [2004.11063] [2010.15781]

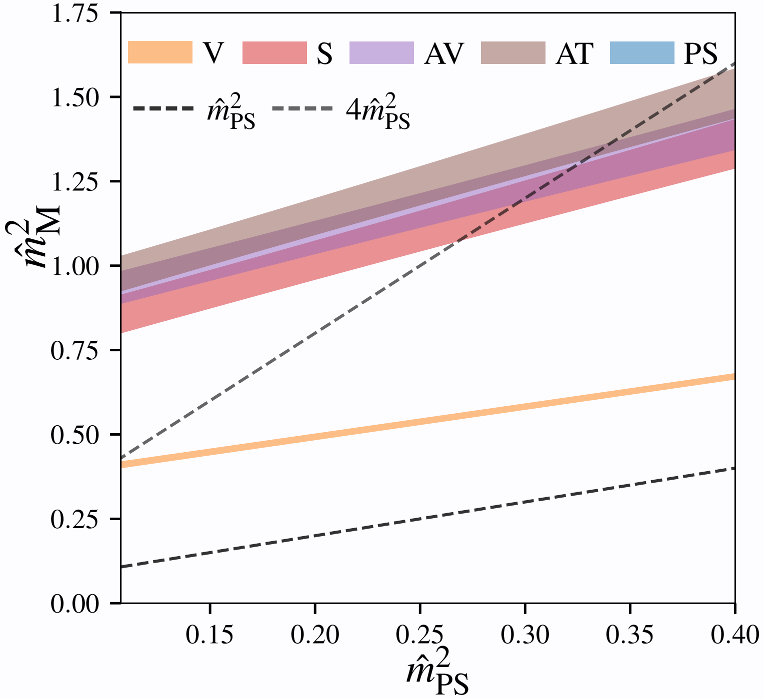
Pure Gauge: Glueball Spectrum

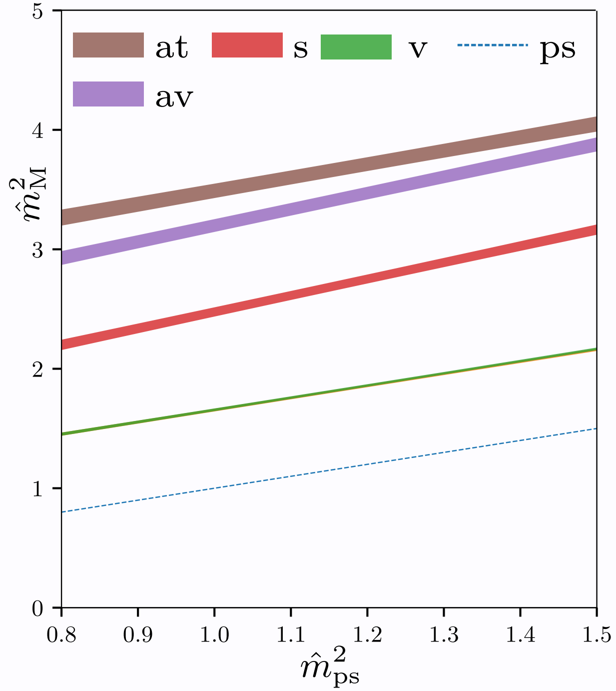


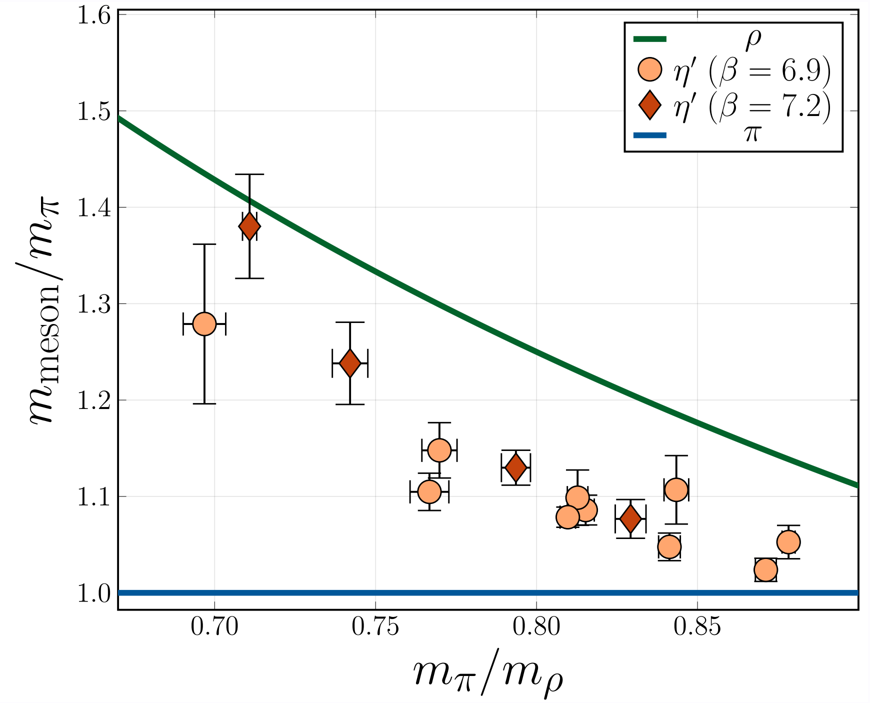
- $N_c = 2N = 2, 4, 6, 8$
- extrapolated to continuum
- compatible with $SU(N_c)$!
- extrapolation $N_c \to \infty$ possible

$Sp(4)_c$ Theories: Single Representation Mesons (FUN, AS)

- Continuum extrapolation for $0^{\pm}, 1^{\pm}$ non-singlet mesons

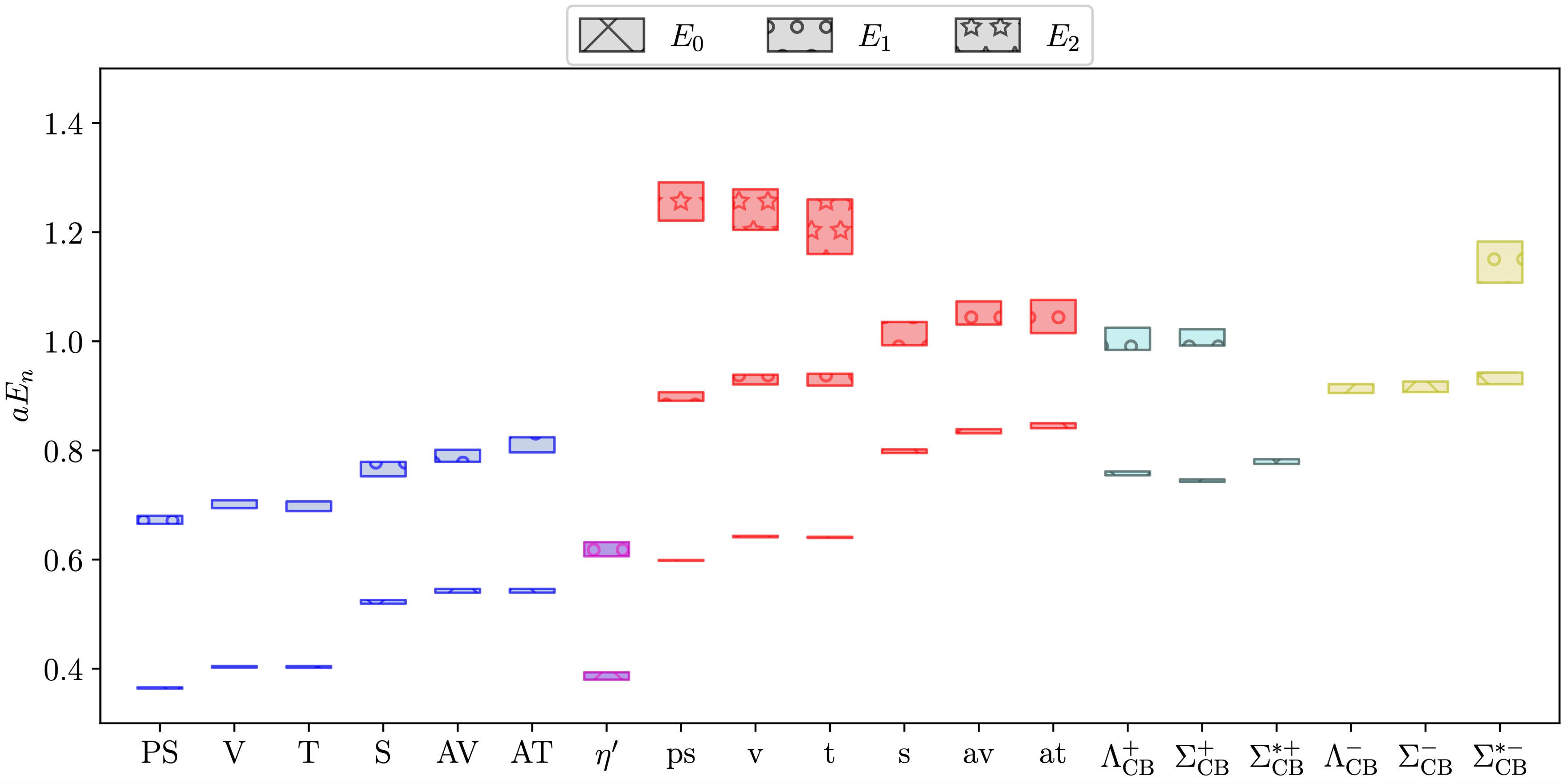






- Fundamental theory: measured 0^- singlet meson mass!

Multi-Representation Theories

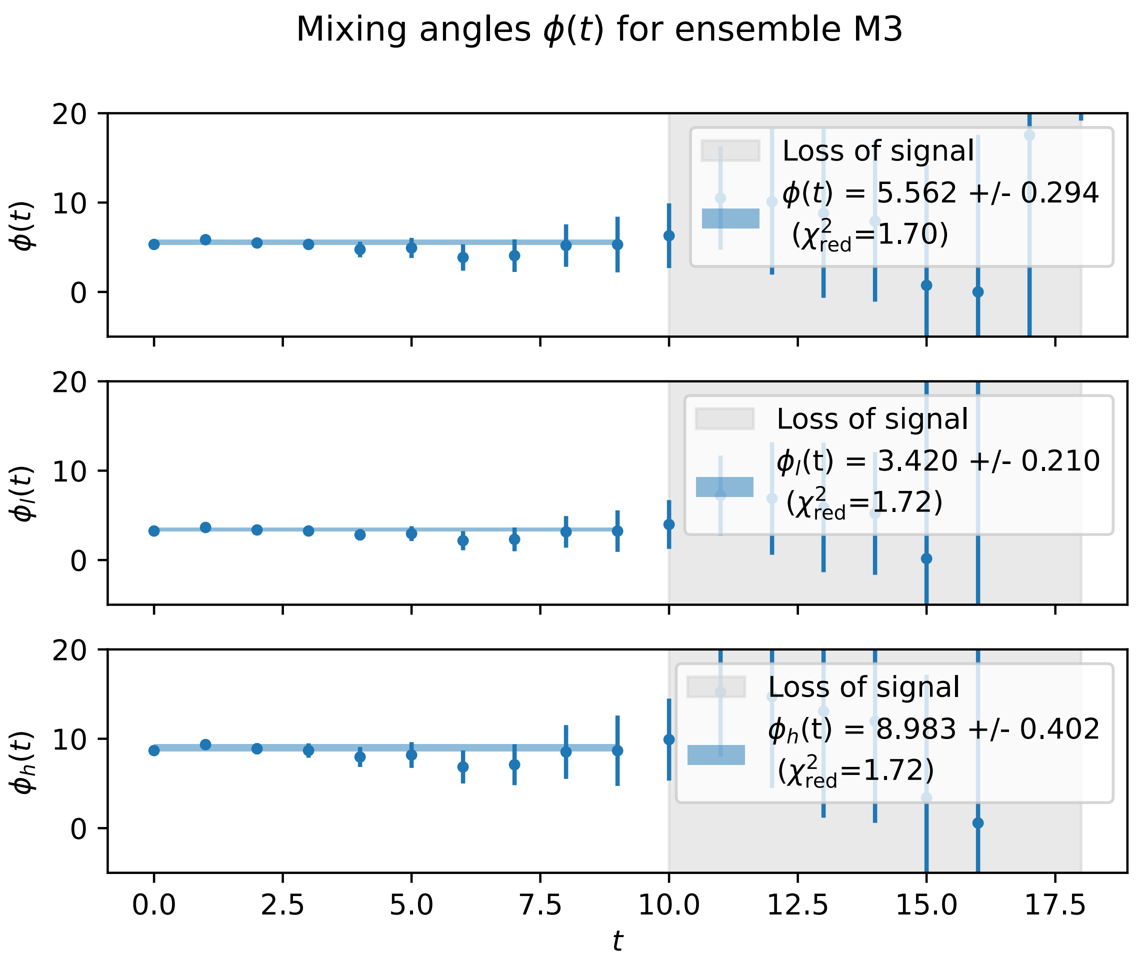


(results for fixed lattice spacing & fermion masses)

contains mesons & baryons including excited states!

Multi-Representation Theories: Mixing

- Mixed theory: Flavour singlet states can mix
- Reminiscent of $\eta - \eta'$ mixing in QCD
  - One Goldstone boson η_l
  - Other state receives $U(1)_A$ contribution η_h
- We can measure their mixing angle ϕ!



Outlook on the Lattice: What else is possible?

- Scattering of Goldstones, resonances, decay widths
- Behaviour at finite temperature, phase transitions
  - 1st order transitions give rise to Gravitational Waves
  - Such transitions are established in Pure Gauge and for heavy fermions
- (Pseudo-)real theories: Finite density (no sign problem!)
  - Has been extensively studied for $SU(2)$ and G_2
- Form factors, decay constants, ...

Future Plans / Work in Progress

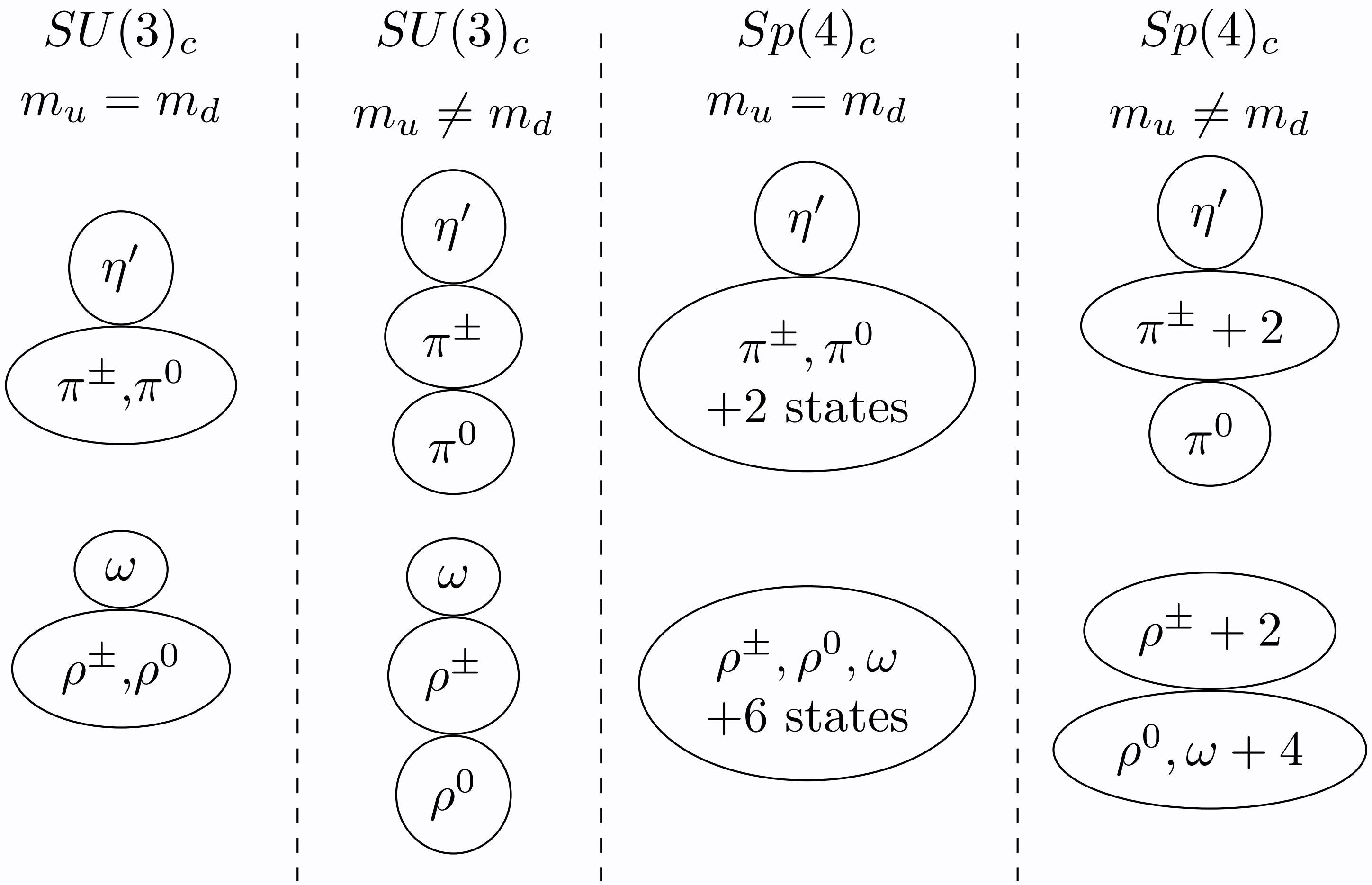
- Resonant Scattering of Goldstone Bosons
- Meson Spectrum at finite T and with $\mathcal{O}(a)$ improvement
- Gravitational Wave Spectrum from Pure Gauge Theory
- Glueballs in Presence of Fermions

Lattice Field Theory can provide many useful insights for BSM physics and model building!

Thank you!

Back-up slides

Extra states: colour singlet-diquarks



- Larger symmetries lead to larger meson multiplets
- Extra states are diquarks! These states are not colour-singlets in QCD!

Extra particles?

- 3 Goldstones like the pions of QCD:
- 2 additional states: quark-quark, antiquark-antiquark
  - π_{qq} with flavour structure $u^T S C \gamma_5 d$
  - and corresponding antiquark-antiquark state

- Note the additional charge conjugation operator C
- Flavour symmetry mixes chiral components
- Consequences for **parity of diquarks!**

Alternative parity transformation

$$D : \psi(x,t) \to \pm i\gamma_0\psi(-x,t)$$

- Still a symmetry of the system ($\mathcal{L}$ invariant)
- Commutes with flavour transforms **[1710.07218]**

$\Rightarrow$ same D-parity in every multiplet

- all *Goldstones* are *pseudoscalar*
- all members of ρ-multiplet are *vectors*

Goldstone bosons and parity

name	operator	J^P	J^D
π^-	$\bar{u}\gamma_5 d$	0^-	0^-
π^+	$\bar{d}\gamma_5 u$	0^-	0^-
π^0	$\bar{u}\gamma_5 u - \bar{d}\gamma_5 d$	0^-	0^-
π_{qq}	$u^T SC\gamma_5 d$	0^+	0^-
$\pi_{\bar{q}\bar{q}}$	$\bar{u}SC\gamma_5 \bar{d}^T$	0^+	0^-

- same pattern in all other multiplets, e.g. vector meson multiplet

Finite temperatures on the lattice

$$Z = \int D[A] \exp(-F_{\mu\nu}[A] F_{\mu\nu}[A] / g^2)$$

- Wilson action: links U

$$S[U] = 6V(1 - \underbrace{u_p[U]}_{\text{average plaquette}})$$

temperature $1/T = a(\beta) N_t$

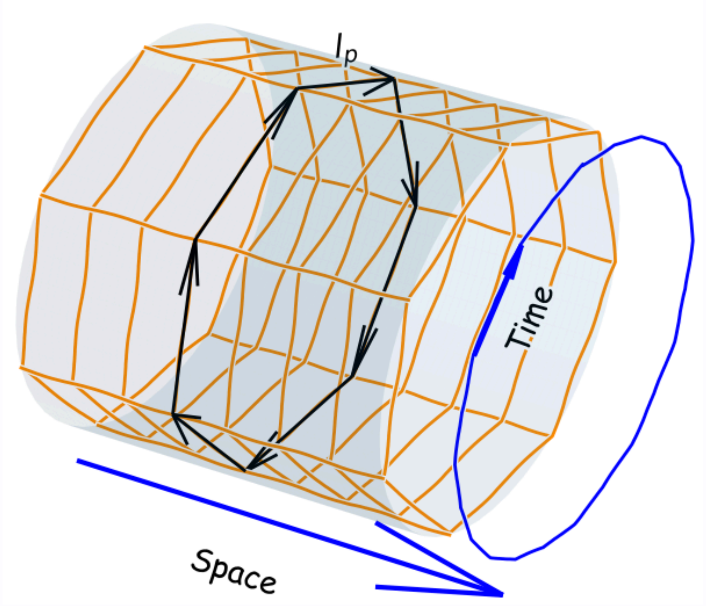


image courtesy of D. Mason

- Relevant Observables: surface tension σ_{cd}, latent heat L_h

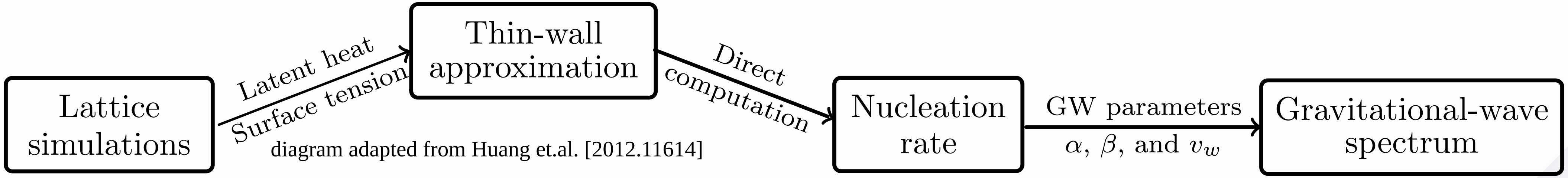


diagram adapted from Huang et.al. [2012.11614]

1st order transitions & importance sampling

- probability $P(E)$ has two peaks (phase coexistence)
- Relates to surface tension σ_{cd} and latent heat L_h

$$\frac{P_{\min}}{P_{\max}} \propto \sqrt{N_s} \exp\left(-2\frac{N_s^2}{N_t^2}\frac{\sigma_{cd}}{T_c^3}\right)$$

$$\Delta\langle u_p\rangle_{\beta_c} \propto L_h/T_c^4$$

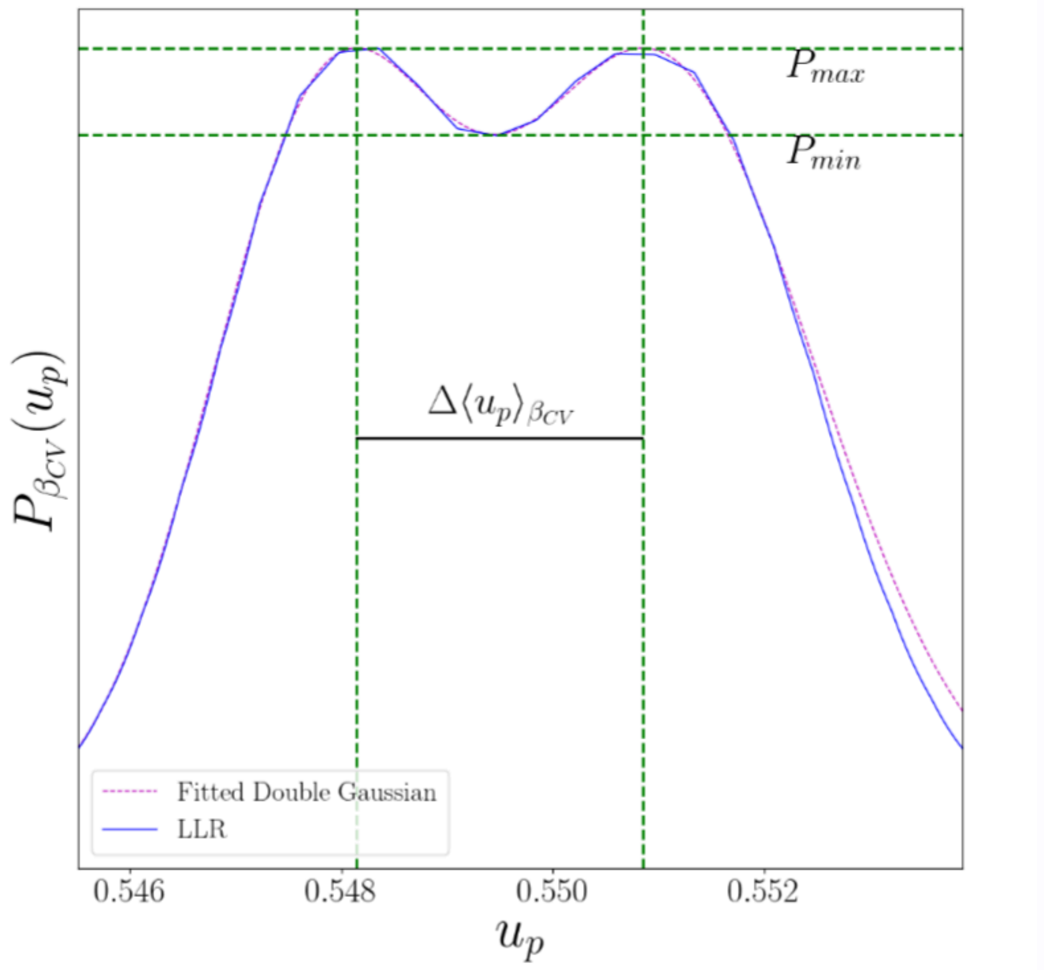
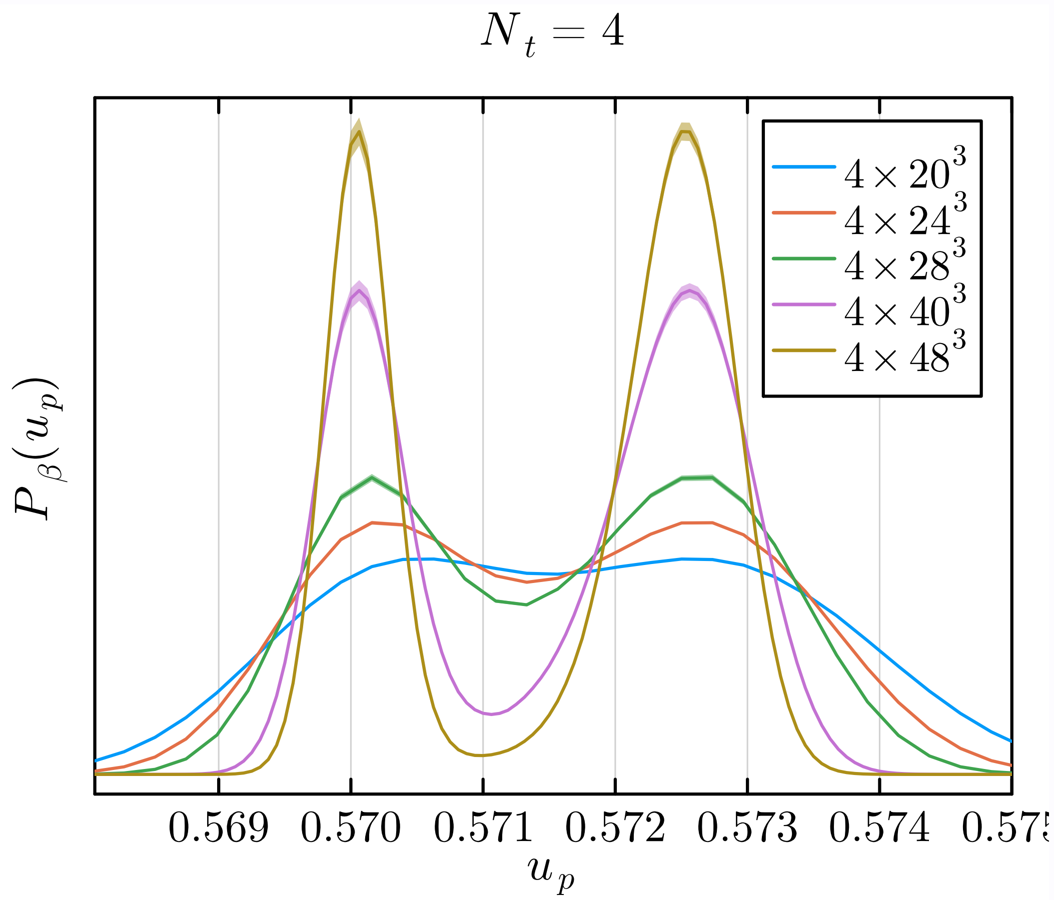


image: thesis D. Mason

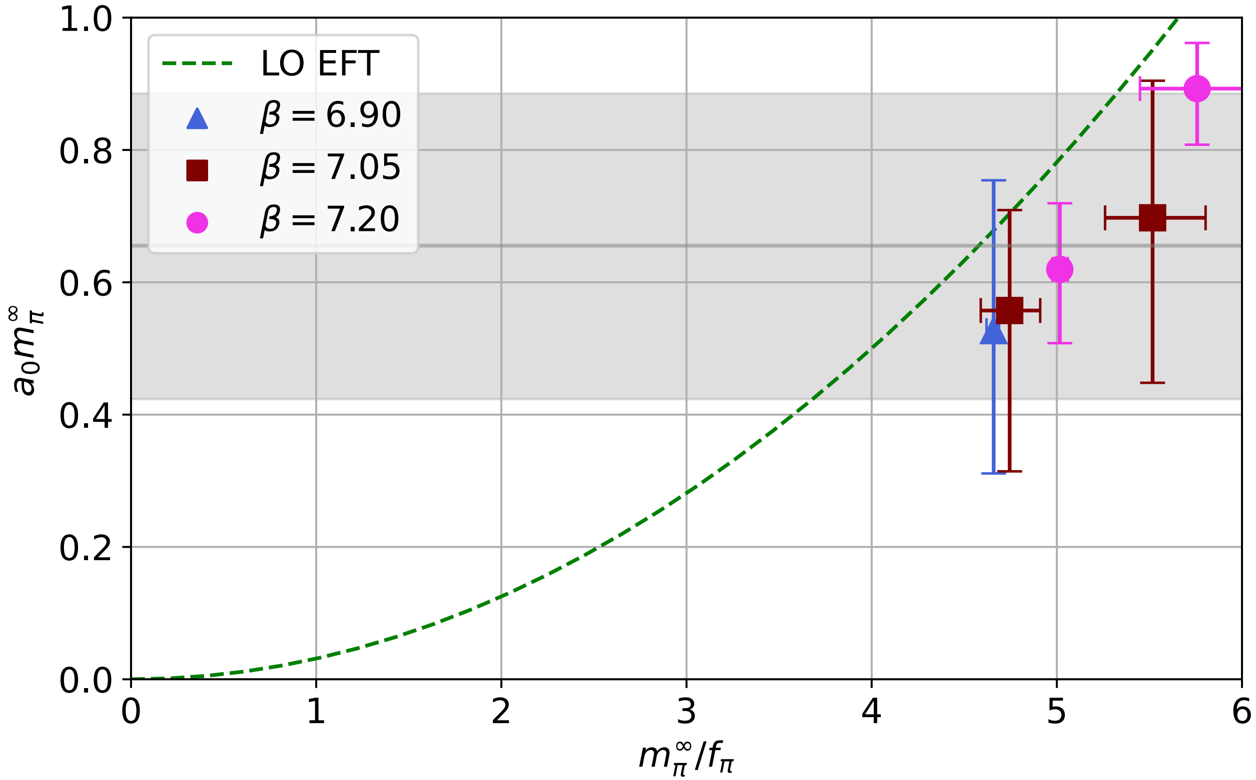
$Sp(4)$ with $N_t = 4$ up to $N_s = 48$



- clear double-peak structure $\Rightarrow$ 1st order!
- β_c determined by requiring equal heights of peaks
- ratio $N_s / N_t > 5$ required
- no clear sign of plateau (=interface) at $N_s / N_t = 12$
- same behaviour at $N_t = 5$

Meson Scattering

$Sp(4)$ with $N_f = 2$



- few lattice energy levels available $\Rightarrow$ systematics
- repulsive, roughly matches χPT
- first step towards other channels and resonances

η' in $N_f = 2$: various gauge groups

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"
- Similar splitting: $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi/m_\rho \rightarrow 0$

mass driven by N_f !