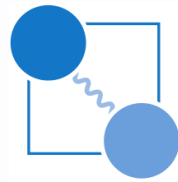


Beyond the Standard Model with Lattice Gauge Theory



T30f
Theoretische Teilchen-
und Kernphysik



Advanced
Grant
EFT-XYZ

Fabian Zierler

(with the TELOS collaboration)



Maier-Leibnitz-Kolloqium, Garching, April 30, 2026

[fzierler.github.io](https://github.com/fzierler)

[teelos-collaboration.github.io](https://github.com/teelos-collaboration)

Overview

1. Strongly Interacting BSM models
 - Strongly Interacting Dark Matter
 - Composite Higgs Models
 - QCD-like Models and Global Symmetries
2. Lattice QCD & Lattice Gauge Theory
3. Lattice Results for BSM models
 - Spectroscopy
 - (Scattering)
 - (Finite temperature behaviour)

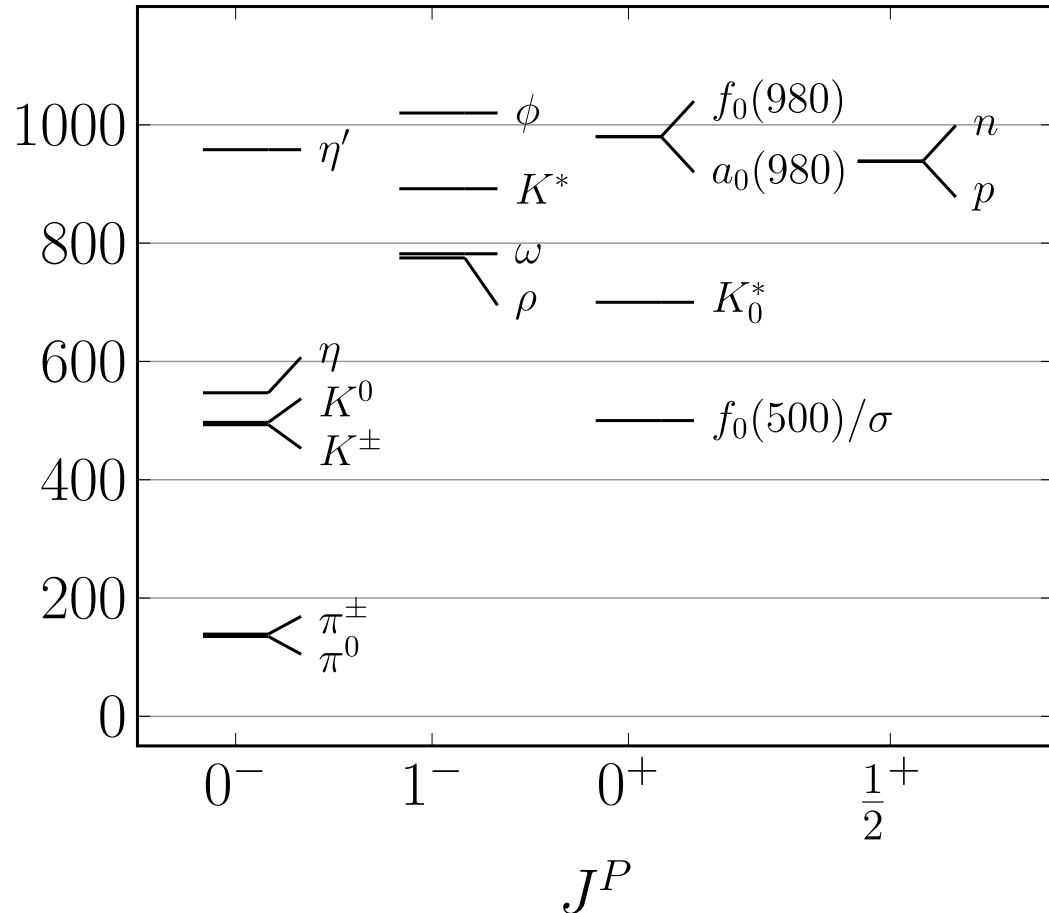
1) Strongly Interacting BSM models

The Standard Model is not complete

- Many other SM extensions exist
- Among them some are **QCD-like** in some sense
- Studied in multiple variations
 - Pure gauge theory: Deconfinement transition, glueballs & large- N behaviour
 - With Fermions: **composite Higgs, composite Dark Matter**
- UV complete due to asymptotic freedom
- This talk: Class of QCD-like BSM models

Spectrum and Symmetries in QCD

Experimental light hadron masses [MeV]



- Set of parametrically light states: π
- Approximate flavour symmetries $SU(2)_F$ from $m_u \approx m_d$
- Lagrangian has larger symmetry for $m_u = m_d = 0$
 \Rightarrow spontaneously broken
 $\Rightarrow \pi$: **pseudo-Goldstone**

Structure of Goldstone BSM extensions

- **Inspired by the symmetries and spectrum of QCD**
- Postulate an additional confining BSM sector
- Spectrum consists of bound states

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}$$
$$\mathcal{L}_{\text{BSM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_f (i\not{D} + m_f) \psi_f + (L_{\text{mediator}})$$

- Contains **parametrically light** bound states:
- *pseudo-Goldstone boson of chiral symmetry breaking*

Global Symmetries of QCD-like Theories

- Flavour/chiral symmetry determined by fermion irrep
 1. complex (e.g. QCD, fundamental $SU(N_c \geq 3), \dots$)
 2. pseudo-real (e.g. fundamental $SU(2), Sp(N_c = 2N), \dots$)
 3. real (e.g. adjoint reps, $SO(N_c), \dots$)
 - (Pseudo-)Real: Different chiral symmetry breaking pattern
 - (Pseudo-)Real: Larger hadronic multiplets
- # of Goldstones: **Dim. of G/H** for breaking pattern $G \rightarrow H$

Unitary and Symplectic Gauge Groups

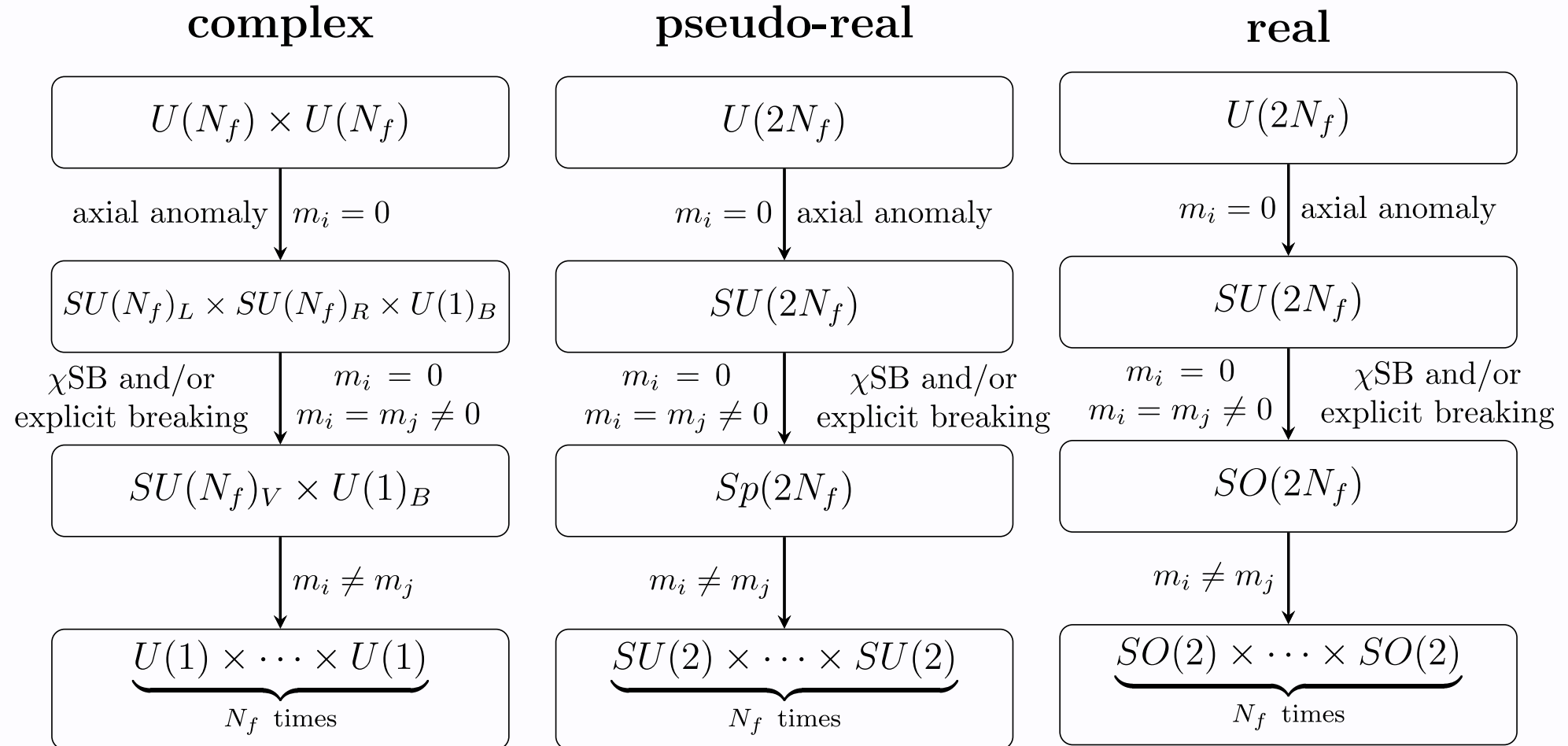
$$\mathcal{L}_{\text{BSM}} = \bar{\psi} \gamma_\mu \underbrace{(\partial_\mu + i A_\mu)}_{D_\mu} \psi + \bar{\psi} M \psi$$

- A_μ from some gauge group G , ψ in an irrep of G
- Complex: Kinetic terms does not mix left & right
- (Pseudo-) Real: Mixing of chiral Weyl components

$$\Psi = \begin{pmatrix} \psi_L \\ -S C \psi_R^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} \quad \begin{array}{l} C \dots \text{charge conj.} \\ S \dots \text{colour matrix} \end{array}$$

$$\mathcal{L}_{\text{BSM}} = i \bar{\Psi} \not{D} \Psi - \frac{1}{2} (\Psi^T S C M \Psi + h.c.) \quad \text{with } S \tau_a S = -\tau_a^T$$

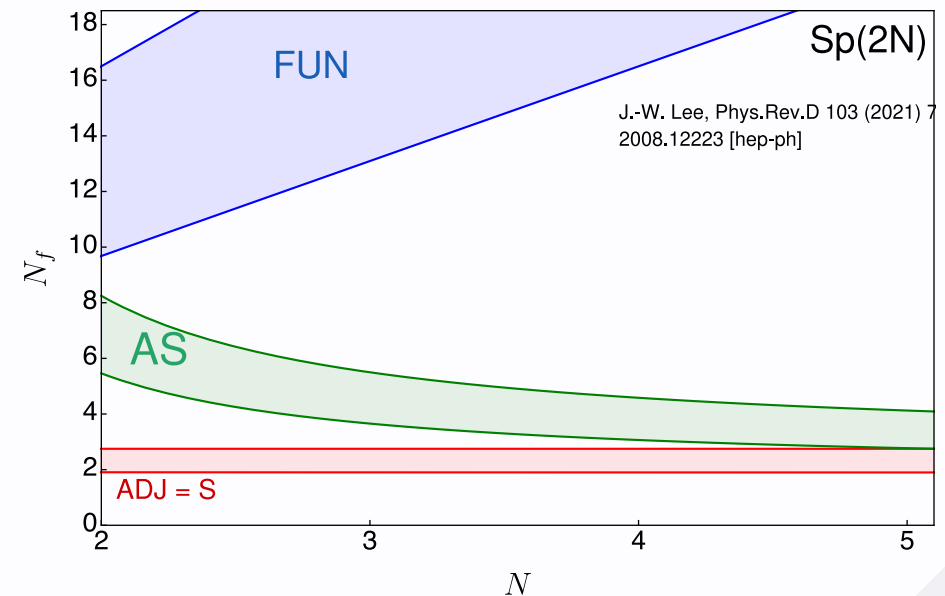
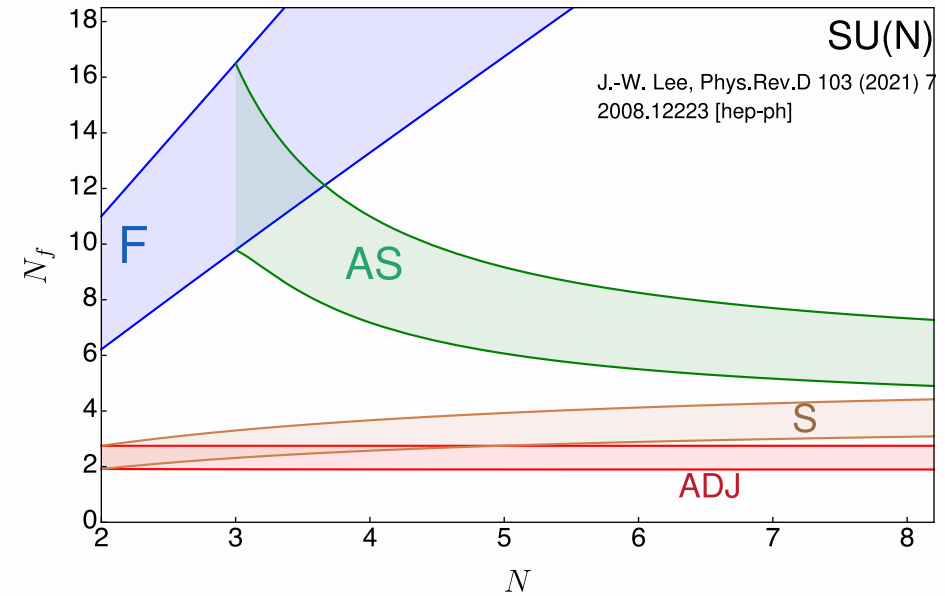
Global Symmetries of QCD-like Theories



Larger global symmetries useful in model building!

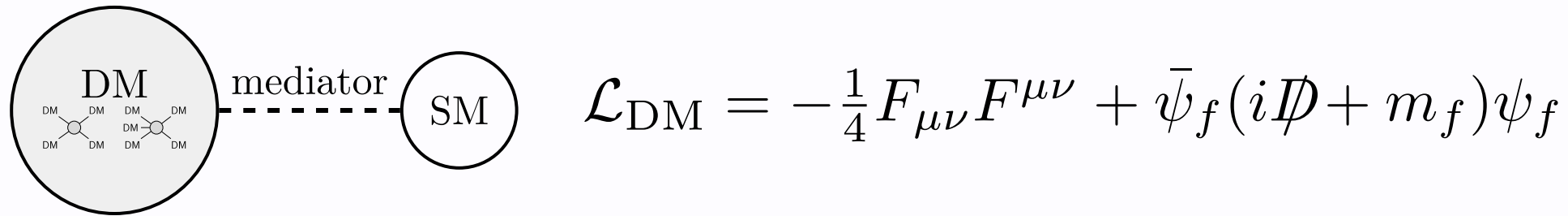
Colour and Flavour Content

- Limit on fermion content
 - asymptotic freedom
 - conformal window
- A priori free to choose:
 - gauge group & N_c
 - fermion irrep & N_f
 - fermion masses m_f
 - gauge coupling $\Leftrightarrow \Lambda_{\text{BSM}}$



Strongly Interacting Gauge Theories in DM Models

- With fermions: Global symmetries make DM stable
- With mediator: Dark sector coupled to SM



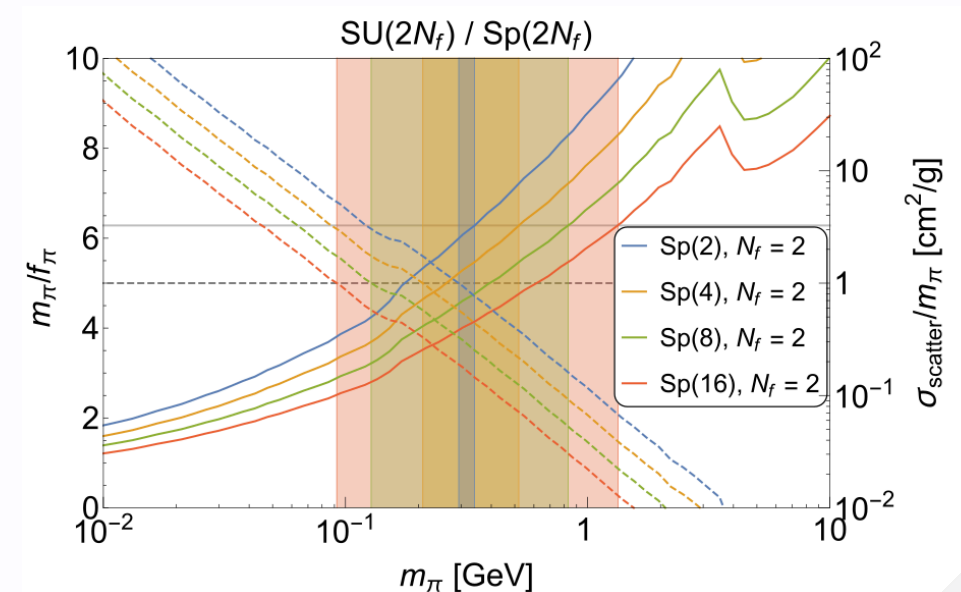
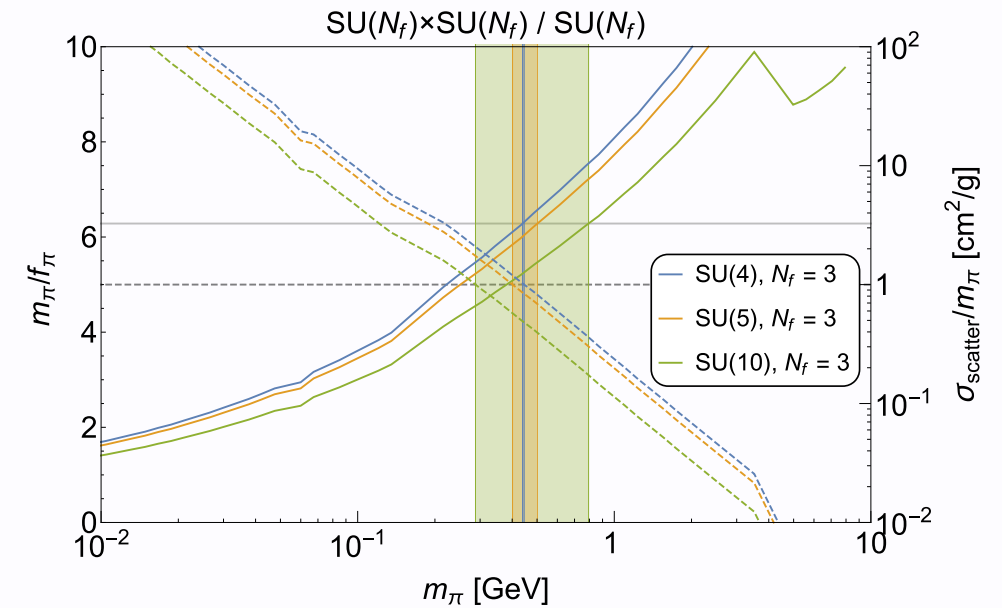
- Dark pions as main Dark Matter candidate
- Non-vanishing self-scattering cross-section arise

$$\langle v \sigma_{\pi_D \pi_D \rightarrow \pi_D \pi_D} \rangle \neq 0$$

- Dark Matter relic density driven by strong processes
- BSM fermion are SM singlets!

Specific Models: Dark Matter

- Example among many:
- **Strongly Interacting Massive Particles** ^[1]
 - DM relic density from $3\pi_D \rightarrow 2\pi_D$ decay
 - in χ PT: Wess-Zumino-Witten term ^[2]
- Almost any coset possible, pseudoreal preferred



Composite Higgs (+ Composite Top): Constraints

- CHM: Higgs boson is composite particle of BSM fermions
- *Partial top compositeness*: top quark composite
- BSM fermions carry SM quantum numbers \Rightarrow few options
 - need to match *all* quantum numbers of Higgs & top in
$$G_{\text{custodial}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$$
 - **requires two distinct** fermion irreps

Composite Higgs + Top Realizations: Possibilities

G_{HC}	ψ	χ	Restrictions	G/H
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\text{HC}} = 7, 9$	$\frac{SU(5)}{SO(5)} \frac{SU(6)}{SO(6)} U(1)$
$SO(N_{\text{HC}})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 7, 9$	
$Sp(2N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\text{HC}} = 4$	$\frac{SU(5)}{SO(5)} \frac{SU(6)}{Sp(6)} U(1)$
$SU(N_{\text{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \bar{\mathbf{F}})$	$N_{\text{HC}} = 4$	$\frac{SU(5)}{SO(5)} \frac{SU(3) \times SU(3)}{SU(3)_D} U(1)$
$SO(N_{\text{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$N_{\text{HC}} = 10$	
$Sp(2N_{\text{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\text{HC}} = 4$	$\frac{SU(4)}{Sp(4)} \frac{SU(6)}{SO(6)} U(1)$
$SO(N_{\text{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 11$	
$SO(N_{\text{HC}})$	$4 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\text{HC}} = 10$	$\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{SO(6)} U(1)$
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\text{HC}} = 4$	
$SU(N_{\text{HC}})$	$4 \times (\mathbf{F}, \bar{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \bar{\mathbf{A}}_2)$	$N_{\text{HC}} = 5, 6$	$\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(3) \times SU(3)}{SU(3)_D} U(1)$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

- Extra Goldstone boson appears due to additional $U(1)^{[1]}$!

Composite Higgs + Top Realizations: Specific Theory

- $Sp(4)_C$ with $N_f = 2$ fundamental and $n_f = 3$ antisymmetric

$$\mathcal{L}_{\text{BSM}} = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^2 \bar{Q}^i (i\not{D} - m_i^f) Q^i + \sum_{j=1}^3 \bar{\Psi}^j (i\not{D} - m_j^{\text{as}}) \Psi^j$$

- Symmetry breaking:

$$SU(4)_F \times SU(6)_{\text{AS}} \times U(1) \rightarrow Sp(4)_F \times SO(6)_{\text{AS}} \times U(1)$$

- Hadron content:

- AS mesons $\bar{\psi}\psi$, F mesons $\bar{Q}Q$, glueballs
- Mixed-representation baryons ("*chimera baryons*")

\Rightarrow composite top partner with $QQ\psi$

Effective Field Theories & Non-Perturbative Physics

- Powerful EFT for Goldstones: Chiral Perturbation Theory!
 - extended to arbitrary gauge group G & fermion rep
 - can be extended to include additional states: ρ, η', \dots
 - has been extended for multiple irreps
- But **requires non-perturbative** input
 - Low energy constants free parameters of EFT
 - EFT has limited applicable energy range!

Goal: Supplement EFTs and go beyond their applicability with non-perturbative calculations from first-principles

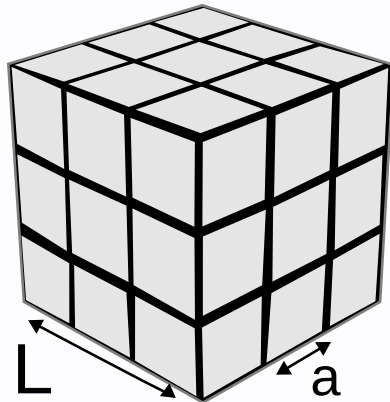
2) Lattice QCD & Lattice Gauge Theory

The Lattice as a Regulator

- *Euclidean* action S on hypercubic lattice

$$Z = \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]}$$

- Discretized spacetime
- lattice spacing a
- volume $V = L^4 = (aN_L)^4$



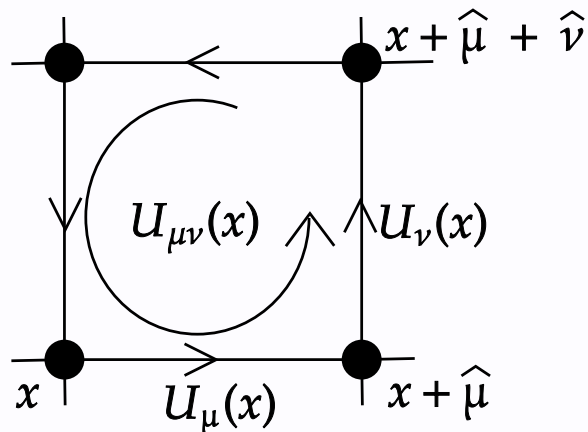
- Lattice regulator
 - UV:** finite **spacing** a
 - IR:** finite **extent** L
- Continuum: $a \rightarrow 0, L \rightarrow \infty$
- Regulator *allows non-perturbative calculations*

Discretized Lattice Action

- Lattice sites \Leftrightarrow fermions, links $U_\mu \Leftrightarrow$ gauge fields
- Lattice action not unique. Here: Standard Wilson action

$$S_g[U_\mu] = \frac{2}{g^2} \sum_{x, \mu < \nu} \text{ReTr} [1 - U_{\mu\nu}(x)] \quad S_f[U_\mu, \psi, \bar{\psi}] = a^4 \sum_{x, y} \bar{\psi}(x) D(x|y) \psi(y)$$

$$D(x|y) = (m + 4/a) \delta_{x, y} - \frac{1}{2a} \sum_{\mu=\pm 1}^{\pm 4} (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu}, y}$$



- Recovers continuum action for limit $a \rightarrow 0$
- Leading corrections $\mathcal{O}(a)$

The Lattice as a Computational Tool

- Solve $\langle O \rangle$ numerically, with Monte-Carlo methods
- Integrate out fermions analytically

$$\begin{aligned} \langle O \rangle &= \frac{1}{Z_g Z_f} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] O[A_\mu, \psi, \bar{\psi}] \exp^{-S_g[A_\mu] - S_f[A_\mu, \psi, \bar{\psi}]} \\ &= \underbrace{\int \frac{1}{Z_g} \mathcal{D}[A_\mu] \det[-D[A_\mu]] \exp^{-S_g[A_\mu]} O_F[A_\mu]}_{\text{interpret as probability distribution } P[A_\mu]} \end{aligned}$$

- Sample gauge fields from $P[A_\mu]$ & evaluate $O_F[A_\mu]$
 \Rightarrow allows non-perturbative evaluation of $\langle O \rangle$

Spectroscopy on the Lattice

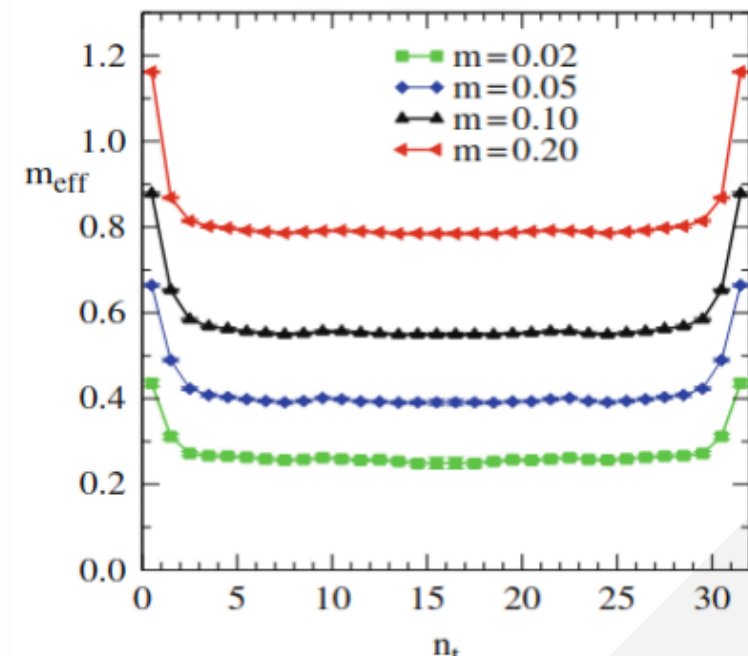
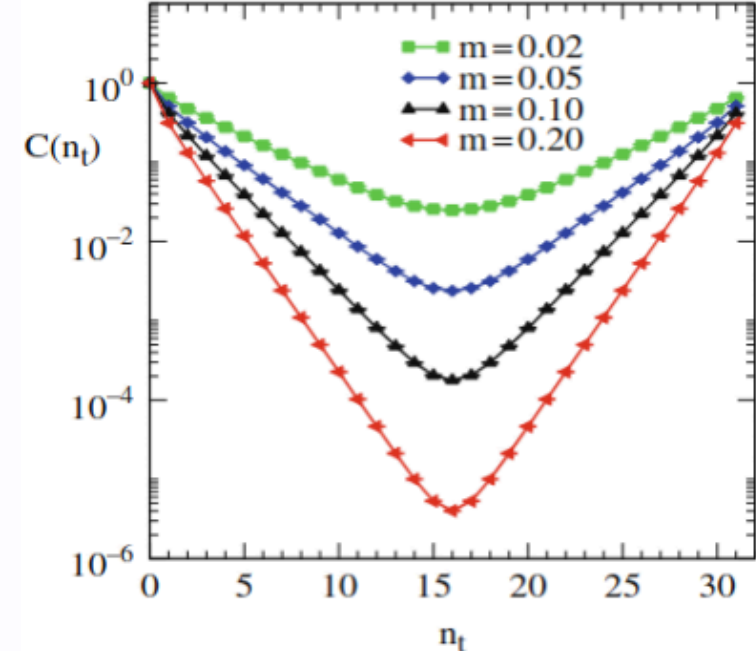
- Operator with desired quantum numbers, e.g. $O_\pi = \bar{u}\gamma_5 d$
- Consider Euclidean correlator $C(t)$

$$C(t = \tau - t') = \sum_{\vec{x}, \vec{y}} \langle O(\vec{x}, \tau) O^\dagger(\vec{y}, t') \rangle$$

$$= \sum_{\vec{x}, \vec{y}, n} \langle 0 | O(\vec{x}, \tau) | n \rangle \langle n | O^\dagger(\vec{y}, t') | 0 \rangle \frac{e^{-E_n t}}{2E_n}$$

$$C(t = \tau - t') = A e^{-E_0 t} + \mathcal{O}(e^{-\Delta E t})$$























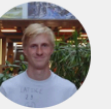










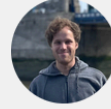


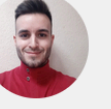





- Energy levels E_n accessible!



3) Results from the TELOS collaboration

Teos Collaboration

Theoretical Explorations on the Lattice with Orthogonal and Symplectic groups.

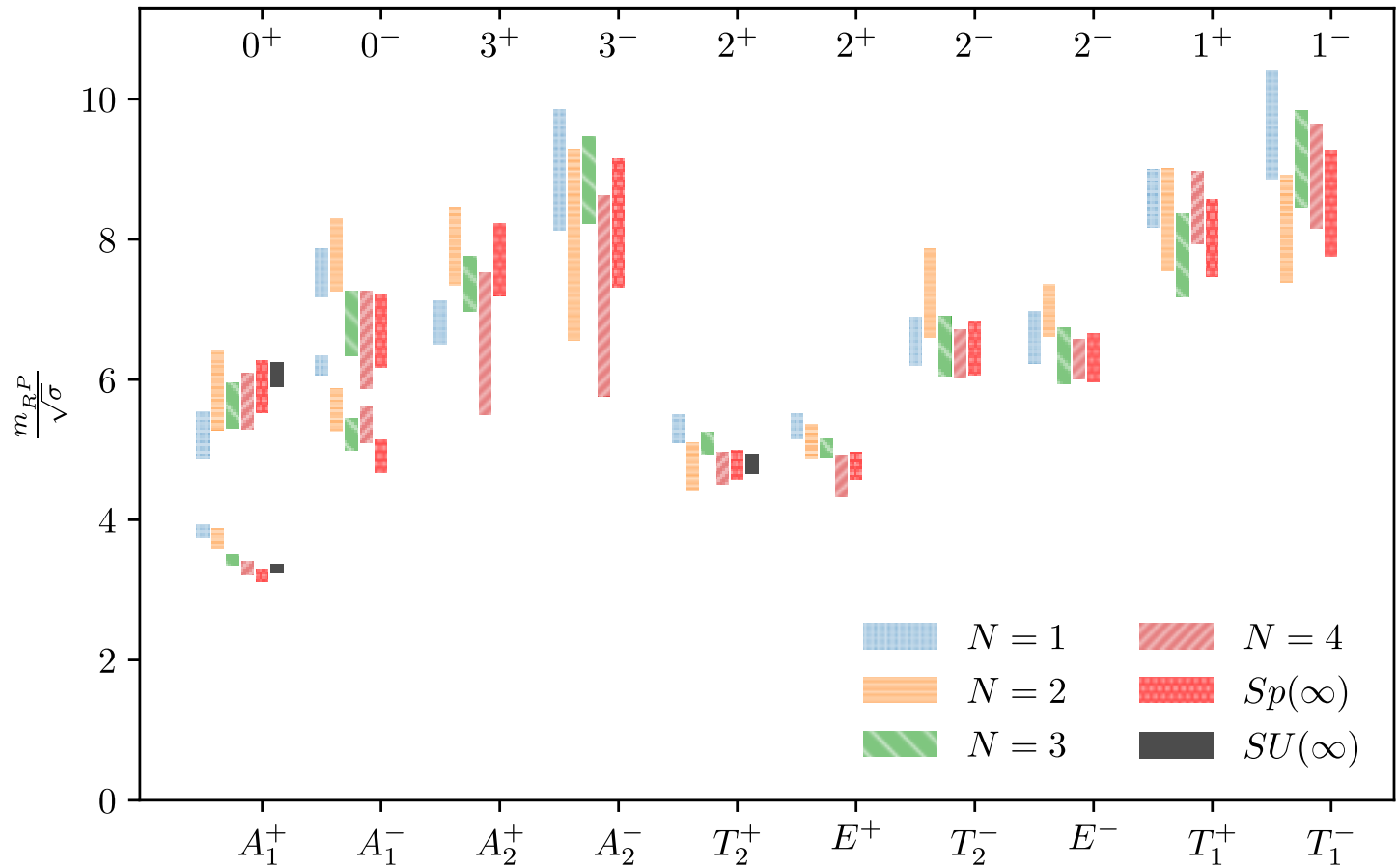
 <p>Biagio Lucini  <i>he/him</i> Swansea University  blucini</p>	 <p>David Lin  National Yang Ming Chiao Tung University</p>	 <p>Davide Vadacchino  <i>he/him</i> University of Plymouth  dvadacchino</p>	 <p>Luigi Del Debbio  University of Edinburgh</p>	 <p>Maurizio Piai  <i>he/him</i> Swansea University  mauriziopai</p>
 <p>Deog Ki Hong  <i>he/him</i> Pusan National University  deogki</p>	 <p>Ed Bennett  <i>they/them</i> Swansea University  edbennett</p>	 <p>Jong-Wan Lee  <i>he/him</i> Institute for Basic Science  jwlee823</p>	 <p>David Mason  <i>he/him</i> Swansea University</p>	 <p>Fabian Zierler  <i>he/him</i> Swansea University  fzierler</p>
 <p>Alexis Verney-Provatas  <i>he/him</i> Swansea University  vataspro</p>	 <p>Gianmarco Simonetti  <i>he/him</i> University of Edinburgh  g-simonetti</p>	 <p>Nicolò Forzano  <i>he/him</i> Swansea University  nickforce989</p>	 <p>Nuno Brito  <i>he/him</i> University of Plymouth  kzone-dev</p>	 <p>Paul Xiao  University of Tsukuba  theHoHsiao</p>

Other investigations by different collaborations with various gauge groups

- Purge Gauge: $SU(N_c \leq 10)$, $Sp(N_c \leq 8)$, G_2, \dots
- Fundamental irrep: $SU(2, 3, 4)$, $Sp(4)$, G_2 , $SO(4)$, G_2, \dots
- Others: $SU(3)Adj$, $SU(2)Adj$, $SU(3)S$, $Sp(4)AS$, $SU(4)S, \dots$
- Mixed: $SU(2)Adj+F$, $SU(4)S+F$, $Sp(4)AS+F$

- In some cases results with various N_f available

AS: anti-symmetric irrep, S: symmetric irrep, Adj: adjoint irrep

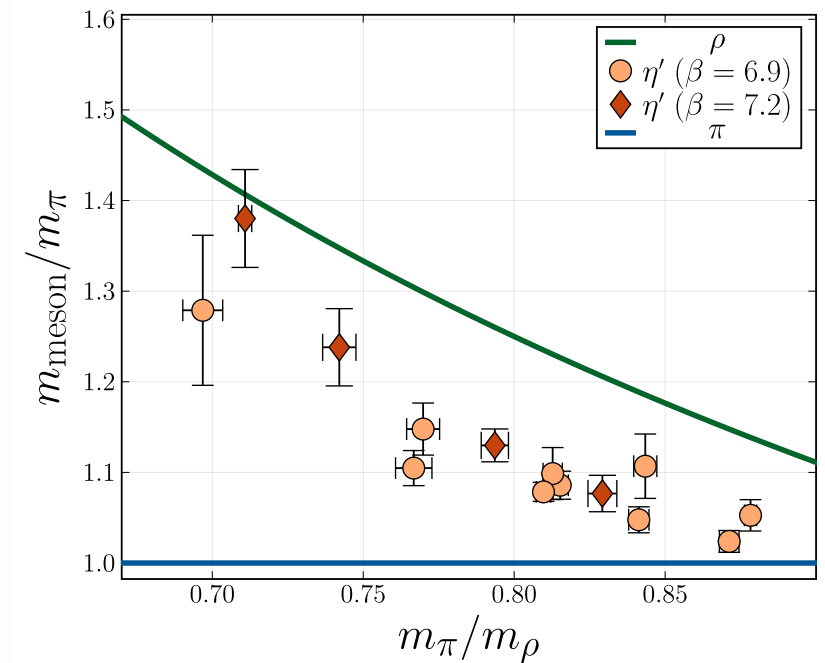
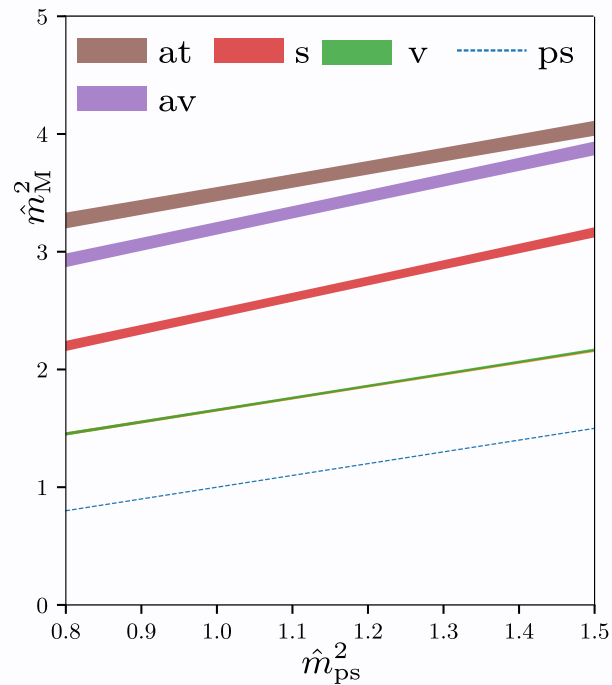
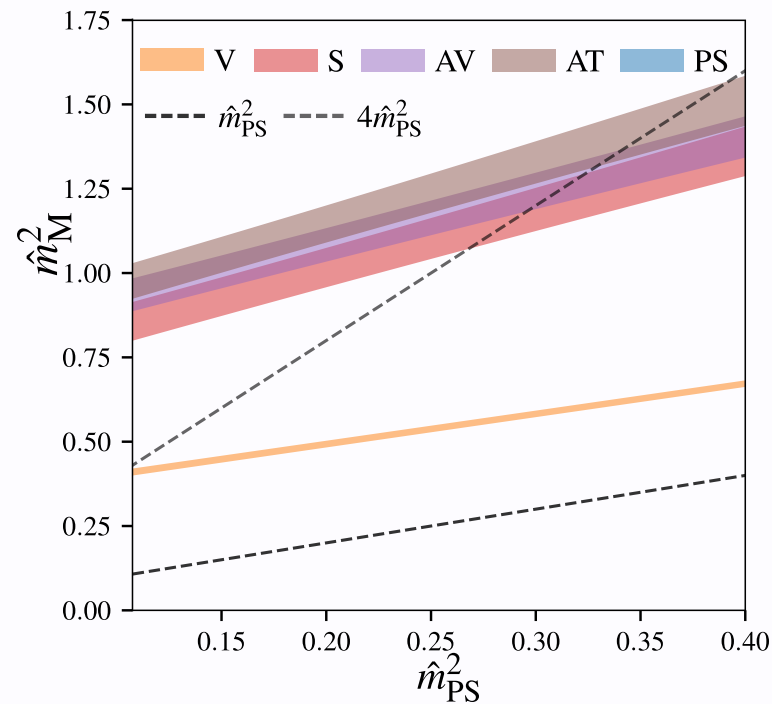


Pure Gauge: Glueball Spectrum

- $N_c = 2N = 2, 4, 6, 8$
- extrapolated to continuum
- similar as $SU(N_c)$
- extrapolation
 $N_c \rightarrow \infty$ possible

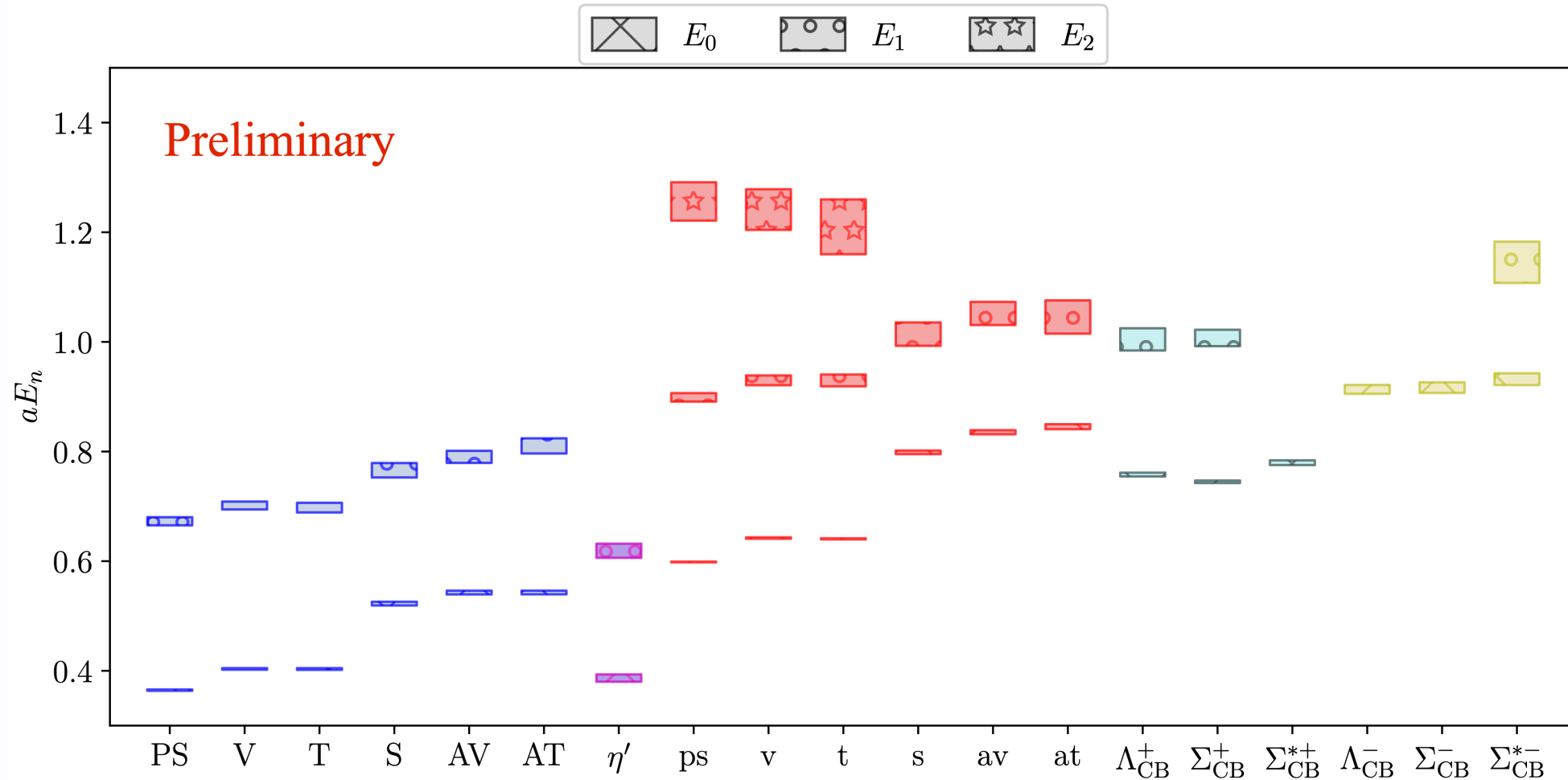
$Sp(4)_c$ Theories: Single Representation Mesons (FUN, AS)

- Continuum extrapolation for $0^\pm, 1^\pm$ non-singlet mesons



- Fundamental theory: measured 0^- singlet meson mass!

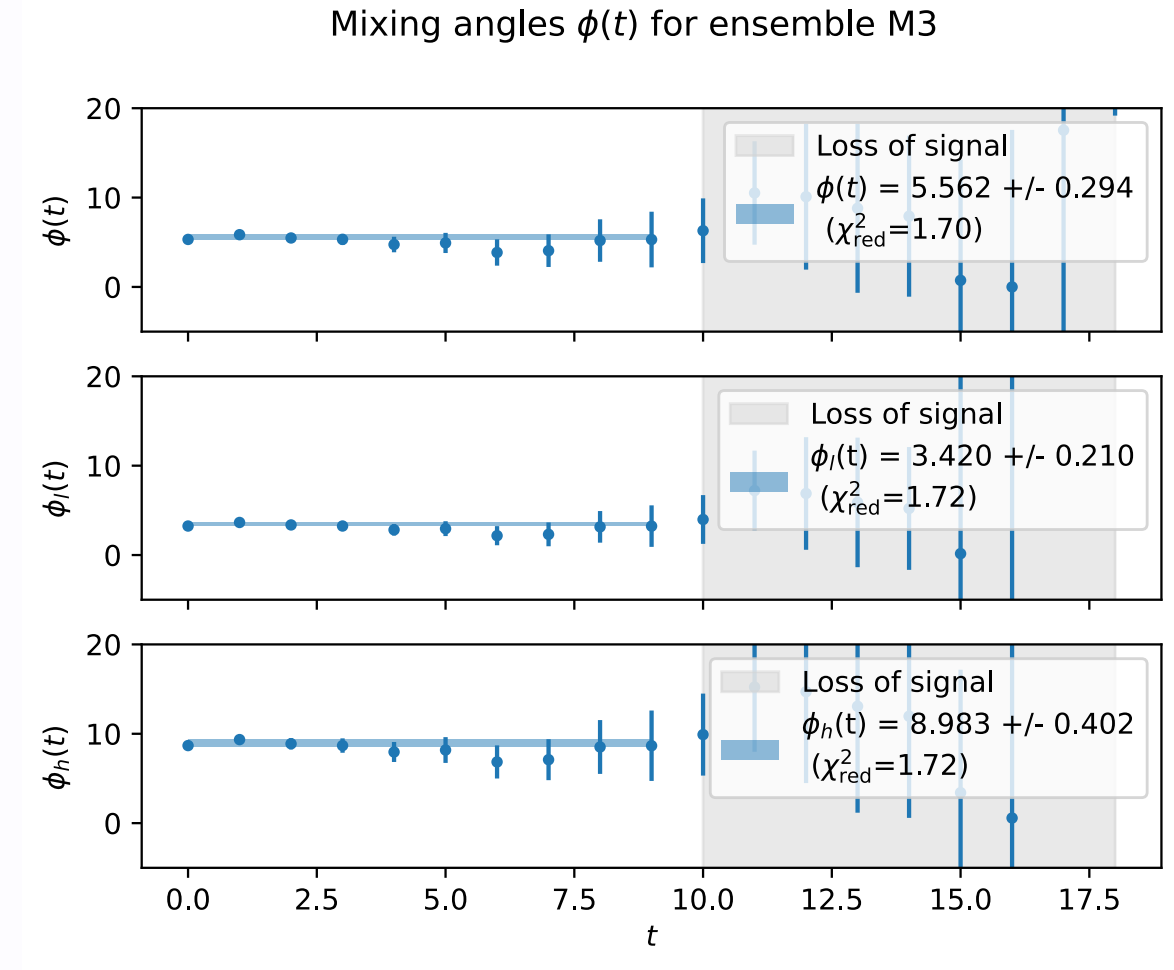
Multi-Representation Theories



contains mesons & baryons including excited states!

Multi-Representation Theories: Mixing

- Mixed theory: Flavour singlet states can mix
- Reminiscent of $\eta - \eta'$ mixing in QCD
 - One Goldstone boson η_l
 - Other state receives $U(1)_A$ contribution η_h
- We can measure their mixing angle ϕ !



Outlook on the Lattice: What else is possible?

- Scattering of Goldstones, resonances, decay widths
- Behaviour at finite temperature, phase transitions
 - 1st order transitions give rise to Gravitational Waves
 - Such transitions are established in Pure Gauge and for heavy fermions
- (Pseudo-)real theories: Finite density (no sign problem!)
 - Has been extensively studied for $SU(2)$ and G_2
- Form factors, decay constants, ...

Future Plans / Work in Progress

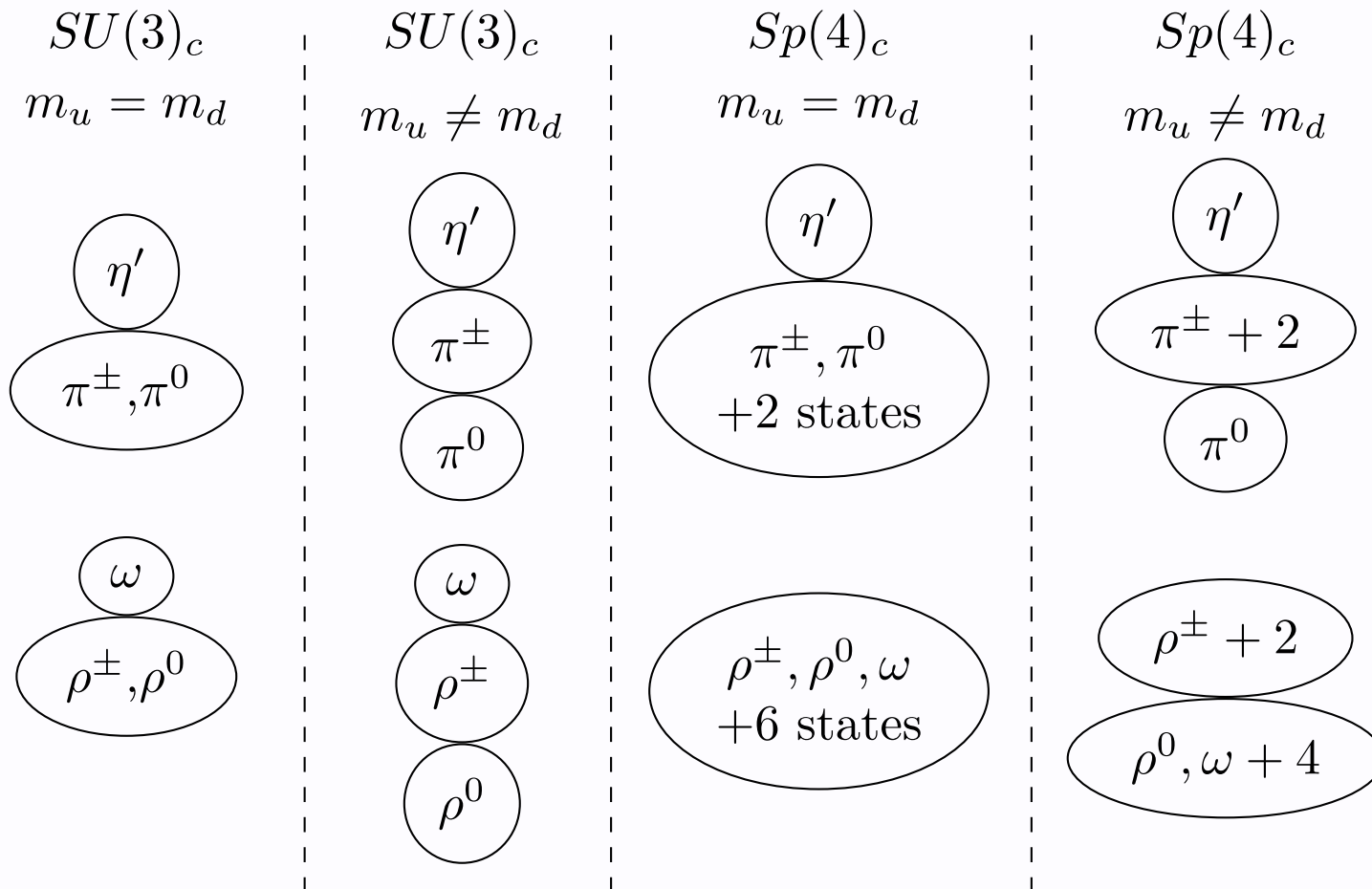
- Resonant Scattering of Goldstone Bosons
- Meson Spectrum at finite T and with $\mathcal{O}(a)$ improvement
- Gravitational Wave Spectrum from Pure Gauge Theory
- Glueballs in Presence of Fermions

**Lattice Field Theory can provide many useful insights for
BSM physics and model building!**

Thank you!

Back-up slides

Extra states: colour singlet-diquarks



- Larger symmetries lead to larger meson multiplets
- Extra states are diquarks! These states are not colour-singlets in QCD!

Finite temperatures on the lattice

$$Z = \int D[A] \exp(-F_{\mu\nu}[A] F_{\mu\nu}[A] / g^2)$$

- Wilson action: links U

$$S[U] = 6V (1 - \underbrace{u_p[U]}_{\text{average plaquette}})$$

temperature $1/T = a(\beta) N_t$

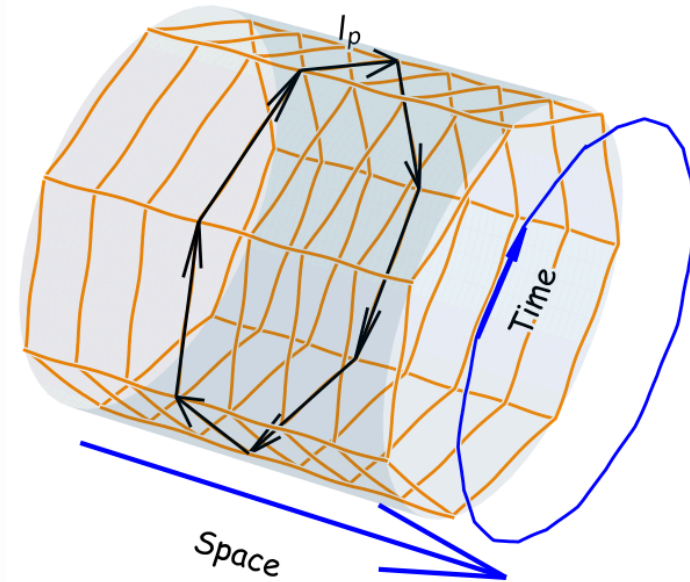
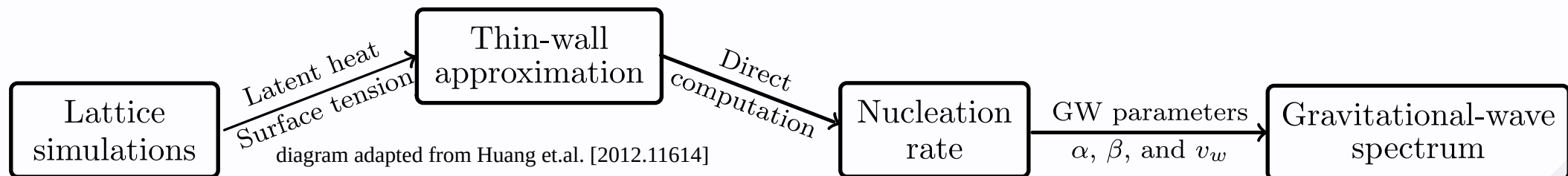


image courtesy of D. Mason

- Relevant Observables: surface tension σ_{cd} , latent heat L_h

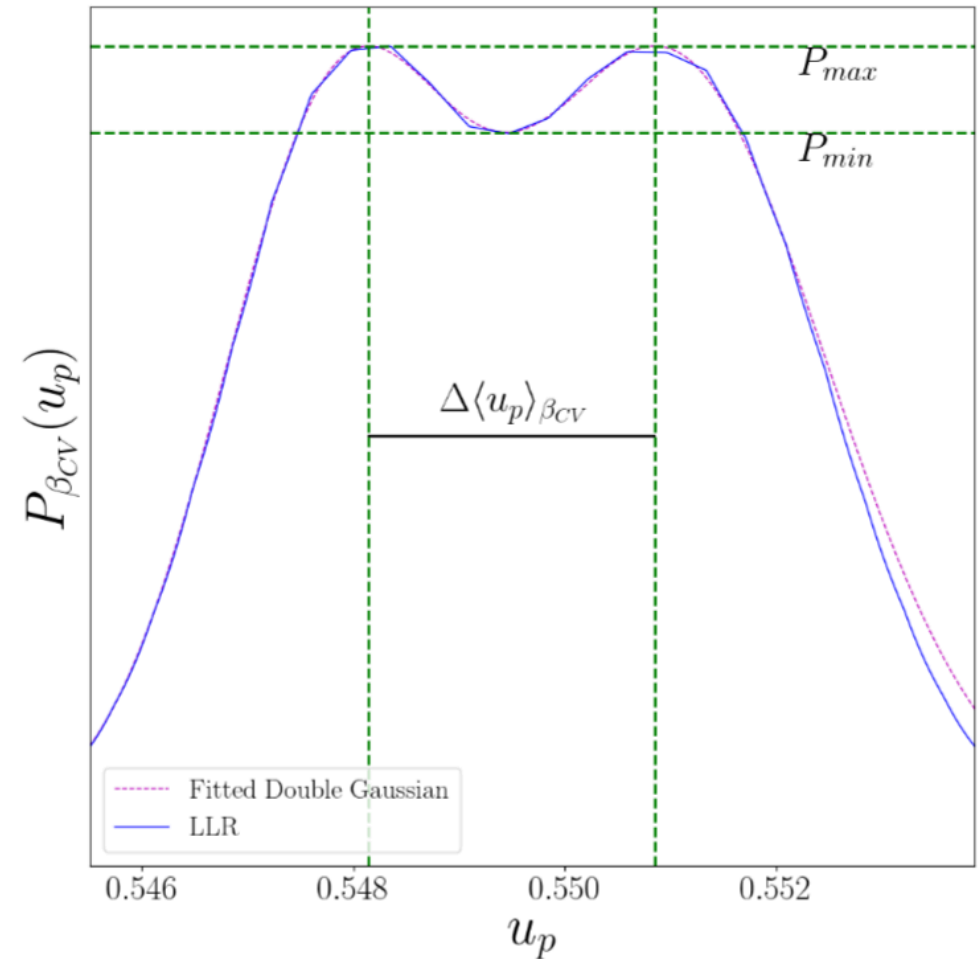


1st order transitions & importance sampling

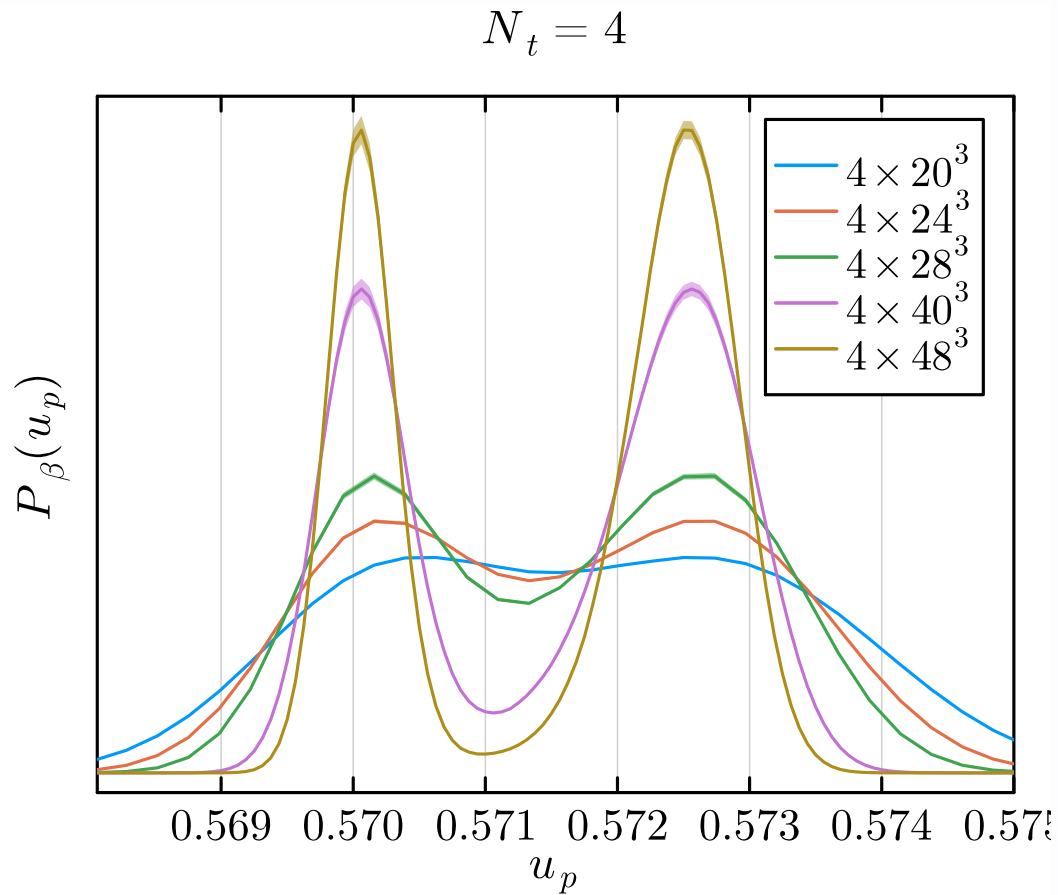
- probability $P(E)$ has two peaks (phase coexistence)
- Relates to surface tension σ_{cd} and latent heat L_h

$$\frac{P_{\min}}{P_{\max}} \propto \sqrt{N_s} \exp\left(-2 \frac{N_s^2}{N_t^2} \frac{\sigma_{cd}}{T_c^3}\right)$$

$$\Delta \langle u_p \rangle_{\beta_c} \propto L_h / T_c^4$$



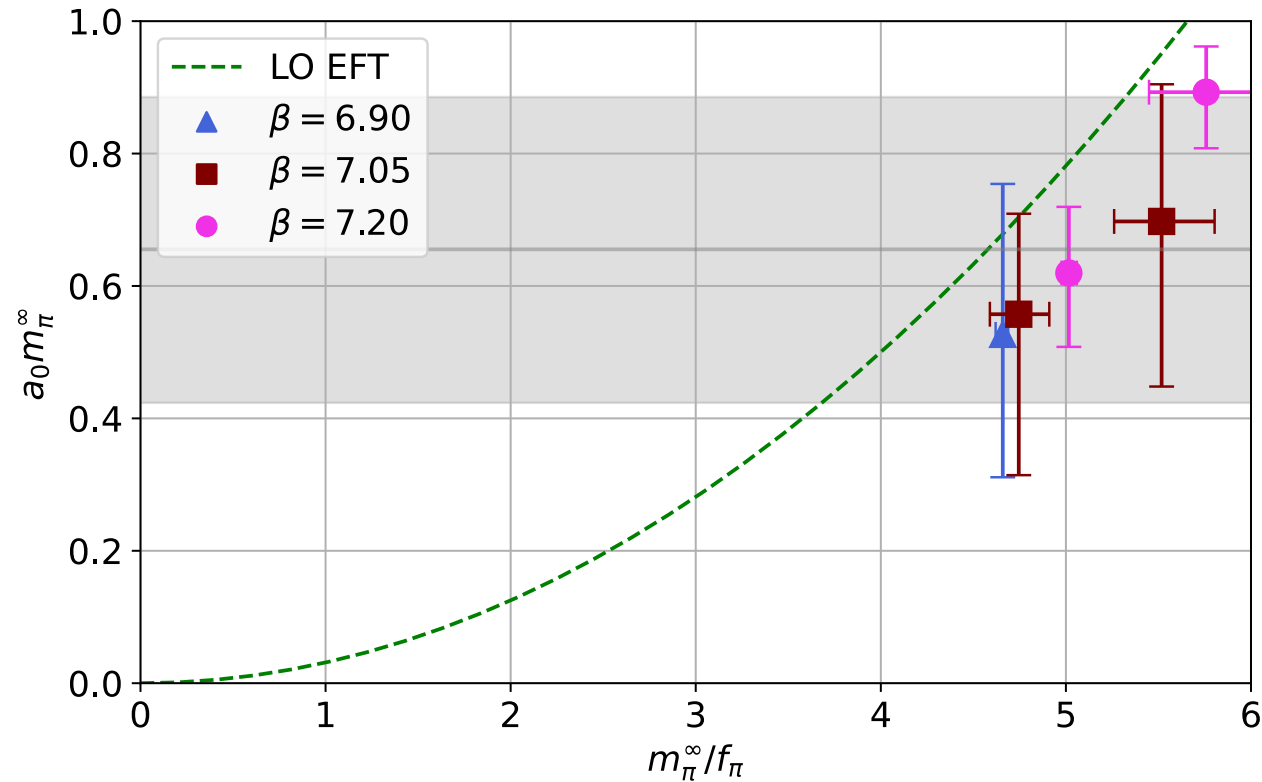
$Sp(4)$ with $N_t = 4$ up to $N_s = 48$



- clear double-peak structure \Rightarrow 1st order!
- β_c determined by requiring equal heights of peaks
- ratio $N_s / N_t > 5$ required
- no clear sign of plateau (=interface) at $N_s / N_t = 12$
- same behaviour at $N_t = 5$

Meson Scattering

$Sp(4)$ with $N_f = 2$



- few lattice energy levels available \Rightarrow systematics
- repulsive, roughly matches χ PT
- first step towards other channels and resonances

η' in $N_f = 2$: various gauge groups

- Large N_c : $m_{\eta'} - m_\pi \propto N_f / N_c$
 - $N_f = 2$ could be "small"
 - $N_c = 4$ could be "large"
- Similar splitting: $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi/m_\rho \rightarrow 0$
mass driven by N_f !

