Composite Dark Matter from Sp(2N) gauge theories

Fabian Zierler

IBS-CTPU, Daejeon, February 28, 2024

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mostly based on 2202.05191, 2304.07191, 2311.18549

slides available at: fzierler.github.io/talks/
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arXiv:2304.07191v

Singlets in gauge theories with fundamental matter

Ed Bennett,[1,*] Ho Hsiao,[2,†] Jong-Wan Lee,[3,4,5,‡] Biagio Lucini,[1,6,§] Axel Maas,[7,¶] Maurizio Piai,[8,**] and Fabian Zierler[7,††]

Scattering of dark pions in an Sp(4) gauge theory

Yannick Dengler,[a,*] Axel Maas[a] and Fabian Zierler[a,b] arXiv:2311.18549

Low-energy effective description of dark $Sp(4)$ theories

arXiv:2202.05191

Suchita Kulkarni,[a] Axel Maas,[a] Seán Mee,[a] Marco Nikolic,[b] Josef Pradler,[b] Fabian Zierler[a]

Singlet Mesons in Dark Sp(4) Theories arXiv:2210.11187v1

Fabian Zierler,[a,*] Jong-Wan Lee,[b,c] Axel Maas[a] and Felix Pressler[a]

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based on work with:
E.Bennett, Y.Dengler, H.Hsiao, S.Kulkarni, JW. Lee, B.Lucini,
A.Maas, S.Mee, M.Nikolic, M.Piai, J.Pradler, F.Pressler
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Outline

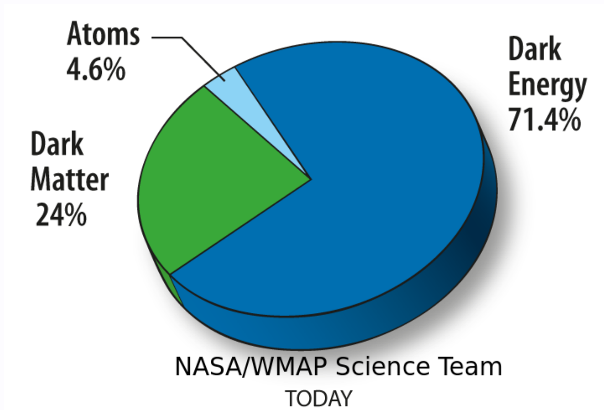
- Composite, self-interacting Dark Matter models
- Strongly Interacting Massive Particles (SIMPs)
- A specific model: $Sp(4)$ with two Dirac fermions
- Lattice Field Theory and numerical results
- Conclusions for Phenomenology and model building

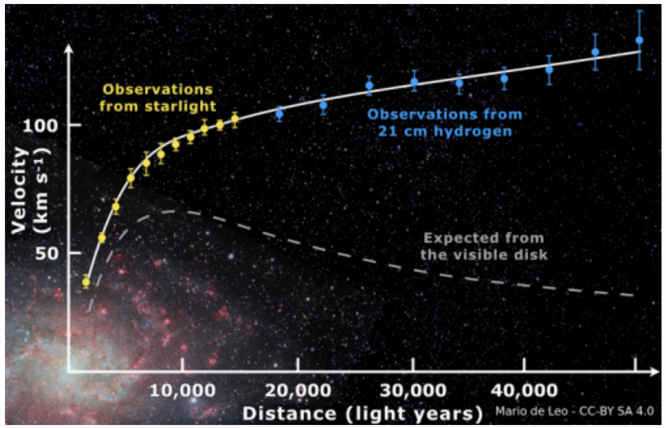
QCD inspired Dark Matter models

[1] e.g. Bertone, Hooper, Silk. [hep-ph/0404175] [2] e.g. PDG review and Famaey, McGaugh [1112.3960]

Dark Matter - Why?

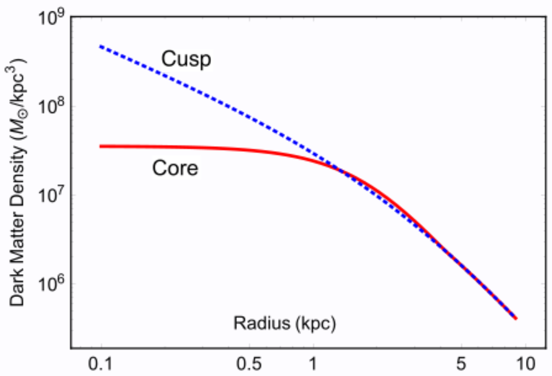
- Strong observational evidence at many scales!
- Modified gravity, primordial black holes are alternatives
- New particles beyond the Standard Model (BSM) promising!

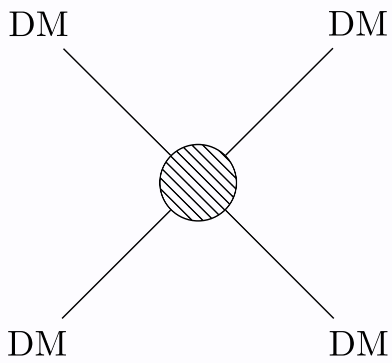


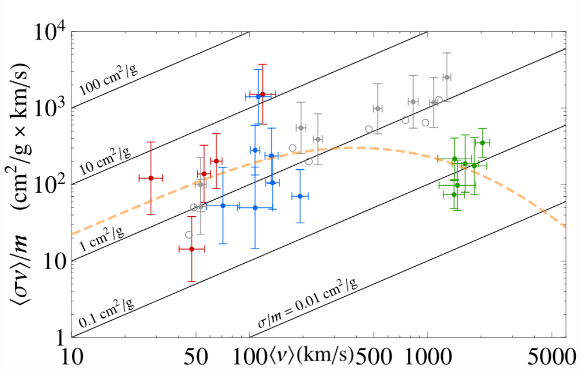


Dark Matter properties

- DM self-interaction phenomenologically allowed[1] and potentially relevant for small-scale structure problems
  - non-vanishing scattering cross-sections $\sigma_{\mathrm{2DM}\to\mathrm{2DM}}$
  - velocity dependence of $\sigma_{\mathrm{2DM}\to\mathrm{2DM}}$ preferred



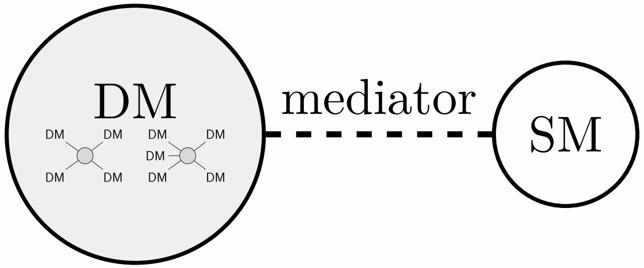




QCD-like Dark Matter can those provide self-interactions!

Strongly Interacting Gauge Theories in DM Models

- With fermions: Global symmetries make DM stable
- With mediator: Dark sector coupled to SM



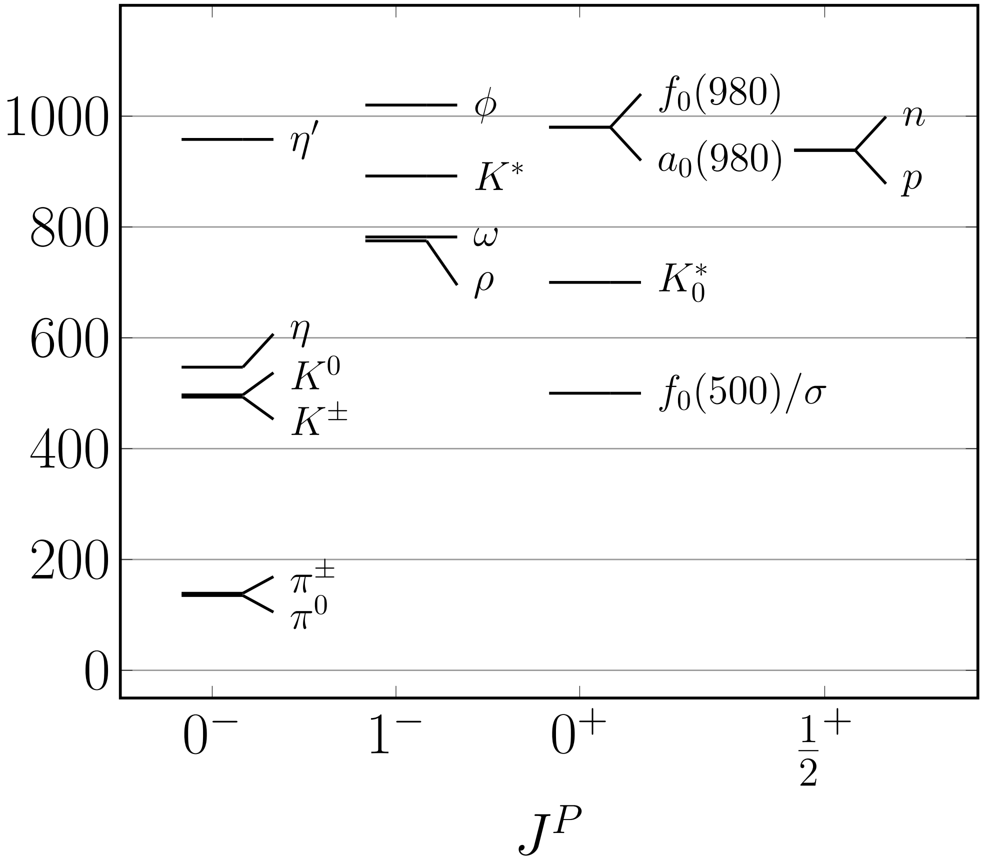
$$\mathcal{L}_{\rm DM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f (i\not{D} + m_f) \psi_f$$

- Non-vanishing self-scattering cross-section arise

$$\langle v\sigma_{\pi\pi\to\pi\pi} \rangle \neq 0$$

- Relic density driven by strong processes

Experimental light hadron masses [MeV]



QCD Spectrum

- π, K, η light: pseudo-Goldstones
- Vectors and scalars light
- Light and broad 0^+ singlet f_0/σ
- Heavy 0^- singlet η'
  $\Rightarrow U(1)_A$ anomalously broken

Dark meson models

- Lightest meson multiplet is protected from decay into SM
  - similar to isospin conservation in QCD
  - symmetry needs to be respected by mediator
- Lightest states: pseudo-Nambu-Goldstone bosons π
  - arise due to chiral symmetry breaking
- have stronger self-interactions than elementary models

Dark meson scattering: Determine DM relic density

- Any model must predict the current density of DM correctly
  - number density n can be calculated using Boltzmann equations

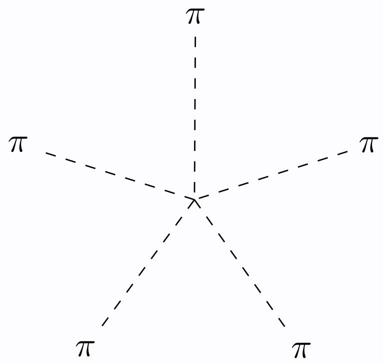
$$\partial_t n + 3Hn = f(\langle v\sigma_{\text{number changing}}\rangle)$$

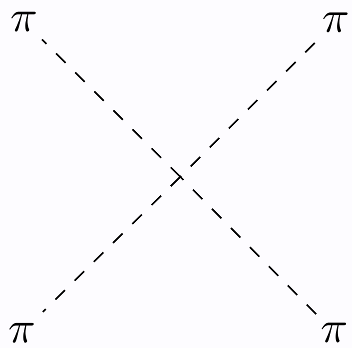
- Cross-sections $\langle \sigma v \rangle$ are input for Boltzmann equations
  - describe non-equilibrium dynamics
  - H is the Hubble rate

[1] Hochberg et. al. [1402.5143] [2] Tulin, Yu [1705.02358], [3] Kondo et.al. [2205.08088] [4] Chu et.al. [2401.12283]

Relevant pion scattering channels

- $3\pi \to 2\pi$ (semi-annihilation) [1]
- $2\pi \to 2\pi$ (self-scattering)
  - self-scattering among DM [2]
  - resonant enhancements [3]
- $2n\pi \to 2\pi$ (multi-hadron bound states) [4]





A concrete model:

Strongly Interacting Massive Particles (SIMPs)

[1] Hochberg et. al. [1402.5143], [2] Wess, Zumino(Phys. Lett. B 1971), Witten (Nucl. Phys. B 1983)

Strongly Interacting Massive Particles (SIMPs)

- Depletion via $3\mathrm{DM} \to 2\mathrm{DM}$ [1], i.e. $3\pi \to 2\pi$
  - same as $KK \to 3\pi$ in QCD [2]
  - Early universe: $\mathrm{SM} \rightleftharpoons \mathrm{DM}$ equilibrium
  - Dark matter depletion process: ***freeze-out***
- LO ChiPT matches relic density at

$$m_\pi \approx \mathcal{O}(100)\mathrm{MeV} - \mathcal{O}(1)\mathrm{GeV}$$

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Dark Matter with 3DM → 2DM depletion and self-interactions
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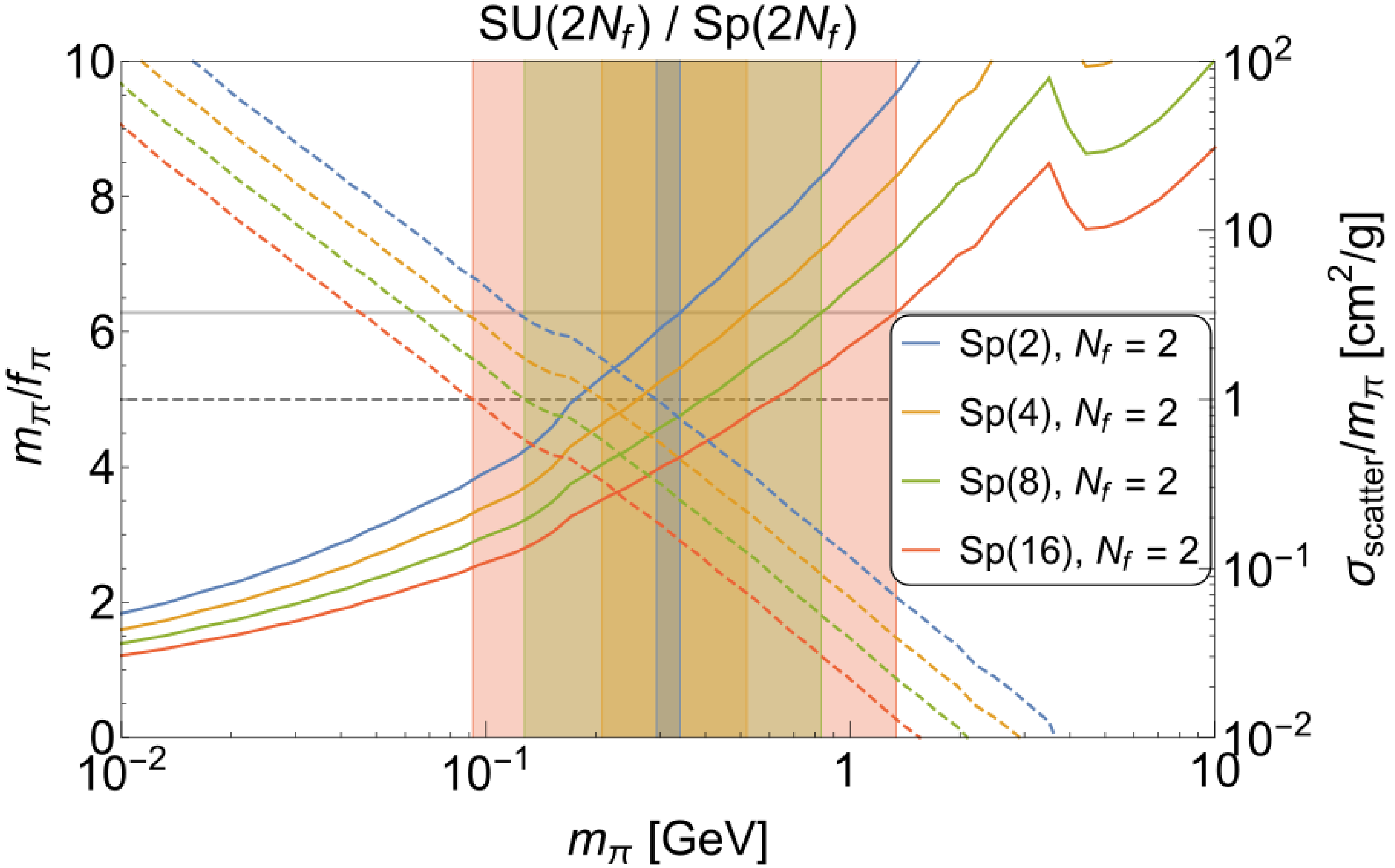
SIMPs at LO ChiPT

Hochberg et. al. [1411.3727]



taken from Hochberg et.al. [1411.3727]

Other mass scales than QCD are relevant!

- Lagrangian has two free parameters: g^2 and m_f
  - one overall energy scale
  - one scale for explicit chiral symmetry breaking
- Overall scale should allow sufficiently heavy DM
- m_f should lead to parametrically light m_π
  - both scales can deviate strongly from QCD!

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Lattice investigations of a larger parameter space are useful!
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[1] Hochberg et. al. [1411.3727] [1512.07917] [2] Choi et.al. [1801.07726] Bernreuther et.al. [2311.17157]
[3] Kulkarni et.al. [2202.05191]

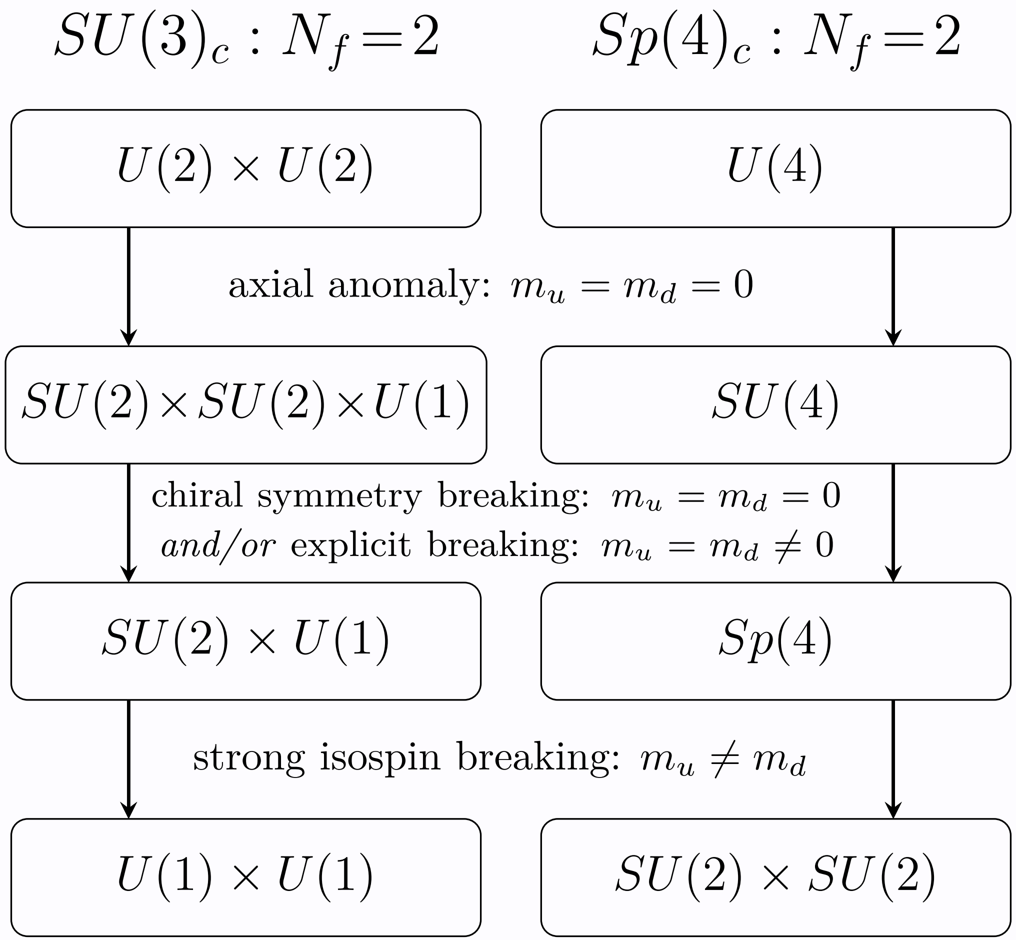
Relevant channels and EFT descriptions

- decay to Standard Model: $2\pi \to SM$ [1]
- involvement of vector mesons: $\pi\pi \to \pi\rho,\, 3\pi \to \pi\rho$ [2]
- influence of light singlets: $\eta'\eta' \to \pi\pi,\, \pi\pi \to \eta'\pi,\, \ldots$ [3]

- The relevance depends on the spectrum
  - investigation of the meson spectrum important
  - lattice investigations inform EFT construction

A concrete model theory:

Two-flavour Sp(4) Gauge Theory



[1] Kosower (Phys.Lett.B. 1984)

[2] Hochberg et. al. [1411.3727] [1512.07917]

SIMPs from $Sp(4)$ gauge theory

- Pseudo-real representation: [1]
  $\Rightarrow$ more pseudo-Goldstones
  $\Rightarrow$ no fermionic bound states
- $N_f = 2$: exactly 5 Goldstones
  - Allows $3\text{DM} \to 2\text{DM}$ [2]

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Sp(4) with two fermions is
a minimal SIMP DM realisation
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Lagrangian of $Sp(4)_c$ with fermions

$$\mathcal{L}_{Sp(4)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f (i\not{D} + m_f)\psi_f$$

- Higher symmetry than QCD-like theories

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SCu_R^* \\ -SCd_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

$C \ldots$ charge conj.
$S \ldots$ colour matrix

$$\mathcal{L}_{Sp(4)} = i\bar{\Psi}\not{D}\Psi - \frac{1}{2}\left(\Psi^T SCM\Psi + h.c.\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- generators τ_a in fundamental repr. : $S\tau_a S = -\tau_a^T$
- mass matrix M proportional to symplectic invariant

Meson multiplets of $Sp(4)_c$ with $N_f = 2$

- $Sp(2N_f)$ flavour symmetry between $2N_f$ Weyl components
- Extra gauge invariant states: $q^T \dots q$ and $\bar{q} \dots \bar{q}^T$

$$Sp(4)_F : \qquad 4 \otimes 4 = 1 \oplus 5 \oplus 10$$

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The global symmetries lead to a richer meson multiplet structure!
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Extra meson states:

Diquarks and Anti-Diquarks

$$\pi_1 : \quad \bar{u}\gamma_5 d$$

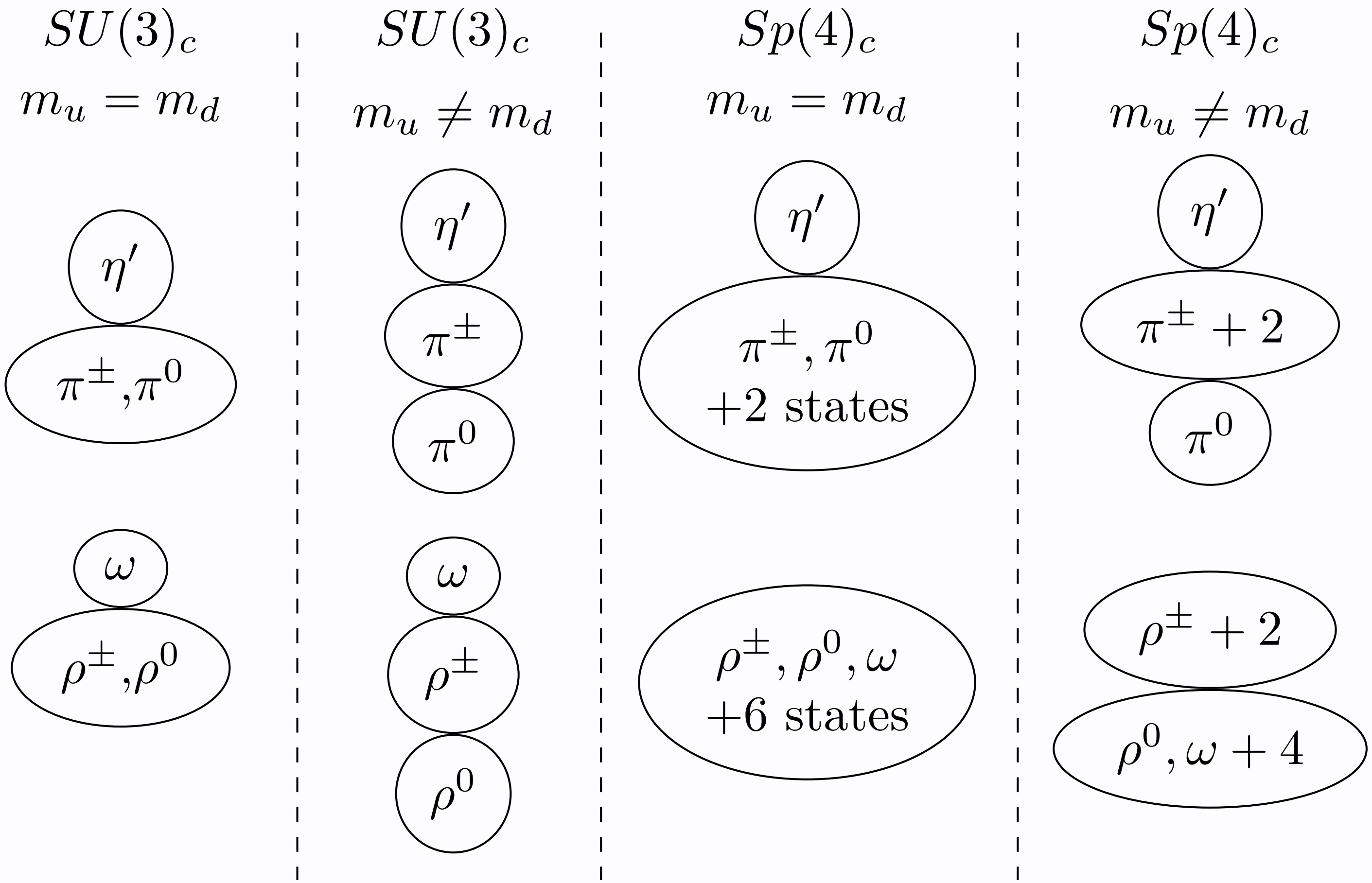
$$\pi_2 : \quad \bar{d}\gamma_5 u$$

$$\pi_3 : \quad \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right)$$

$$\pi_4 : \quad \bar{d}\gamma_5 SC\bar{u}^T$$

$$\pi_5 : \quad d^T SC\gamma_5 u$$

Pseudoscalar (PS) and vector (V) multiplets



The same patterns persist for other channels.

Non-perturbative input is needed:

The case for lattice investigations

The case for lattice investigations

- Theory is non-perturbative at low energies!
  - Lattice allows first-principles calculations
  - Errors are systematically improveable
- Effective field theories are powerful tools!
  - Lattice can calculate low-energy constants
  - provides connection to UV complete theory
- Scattering properties accessible on the lattice!

BSM/DM wishlist from the lattice

1. Masses and decay constants of dark hadrons
   - Non-singlet and singlet mesons, glueballs
2. Scattering of dark pions
   - $2\pi \to 2\pi$ for self-interaction crossection
   - $3\pi \to 2\pi$ for SIMP semi-annihilation
3. Applicability of χPT and related EFTs

Lattice Investigations:

Quantitative Insights

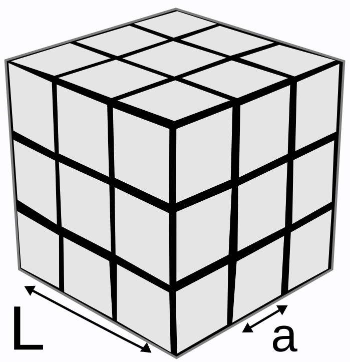
textbooks: e.g. Montvay, Münster 1997 Degrand, Detar 2006 Gattringer, Lang 2010

Lattice setup

- Euclidean action S on hypercubic lattice

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[A_\mu, \psi, \bar{\psi}] e^{-S[A_\mu, \psi, \bar{\psi}]} O[A_\mu, \psi, \bar{\psi}]$$

- Lattice regulator: finite spacing a (UV), finite extent L (IR)



- Calculate observable $\langle O \rangle$ on finite lattice
- Extrapolate to the continuum: $a \to 0$, $L \to \infty$

Lattice spectroscopy: Getting meson masses

- Construct operator with same quantum numbers, e.g.

$$O_\pi = \bar{u}\gamma_5 d \qquad J^P = 0^- (\text{non-singlet})$$

- Spectroscopy for meson from its correlator $C_M(t)$

$$C(t = \tau - t') = \sum_{\vec{x},\vec{y}} \langle O(\vec{x},\tau) O^\dagger(\vec{y},t') \rangle$$

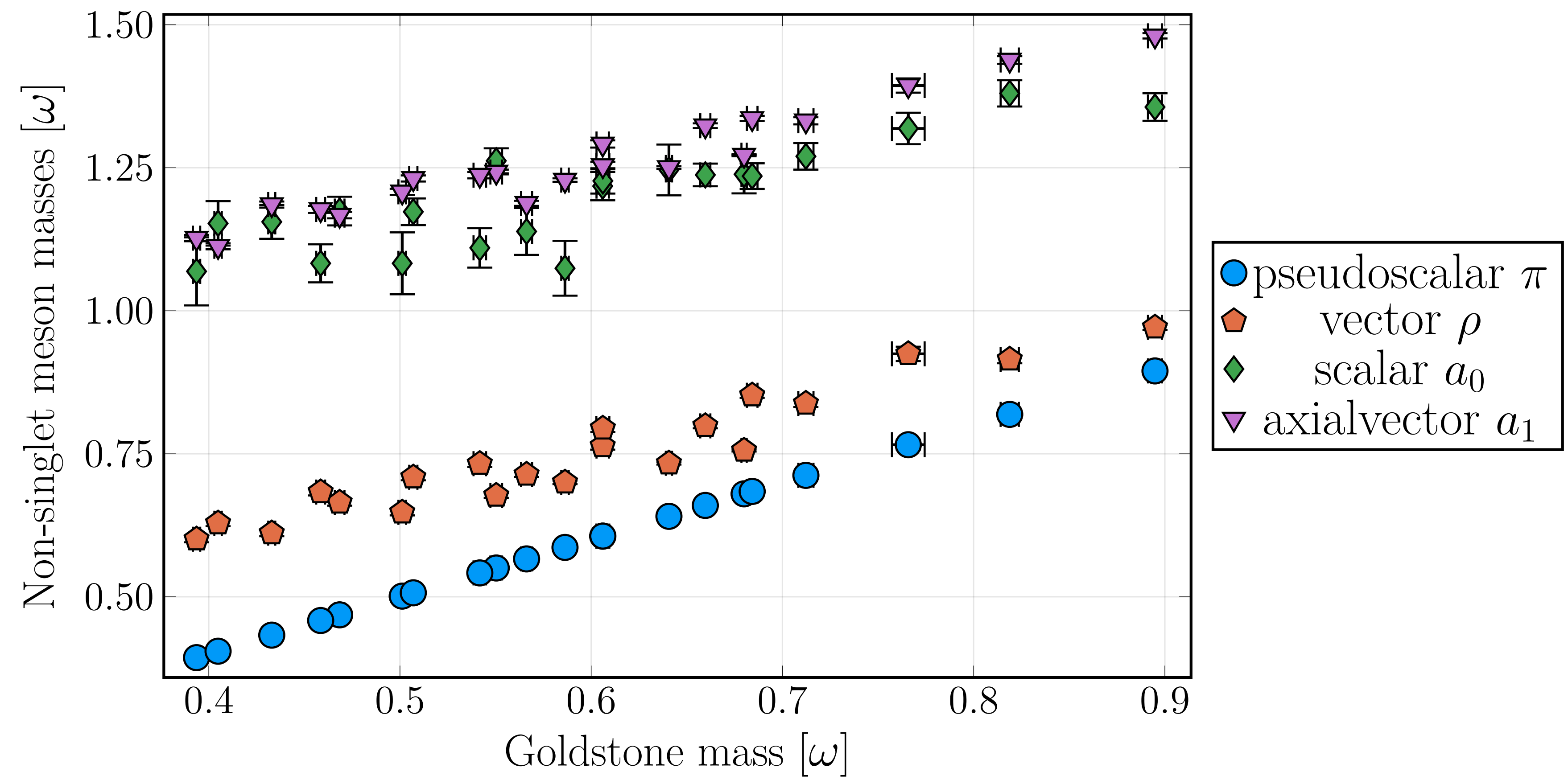
$$= \sum_{\vec{x},\vec{y},n} \langle 0|O(\vec{x},\tau)|n\rangle \langle n|O^\dagger(\vec{y},t')0|\rangle \frac{e^{-E_n t}}{2E_n}$$

$$C(t = \tau - t') = Ae^{-E_0 t} + \mathcal{O}(e^{-\Delta E t})$$

- Ground state mass $E_0 = m$, decay constant $\propto \sqrt{A}$

Non-singlet spectrum

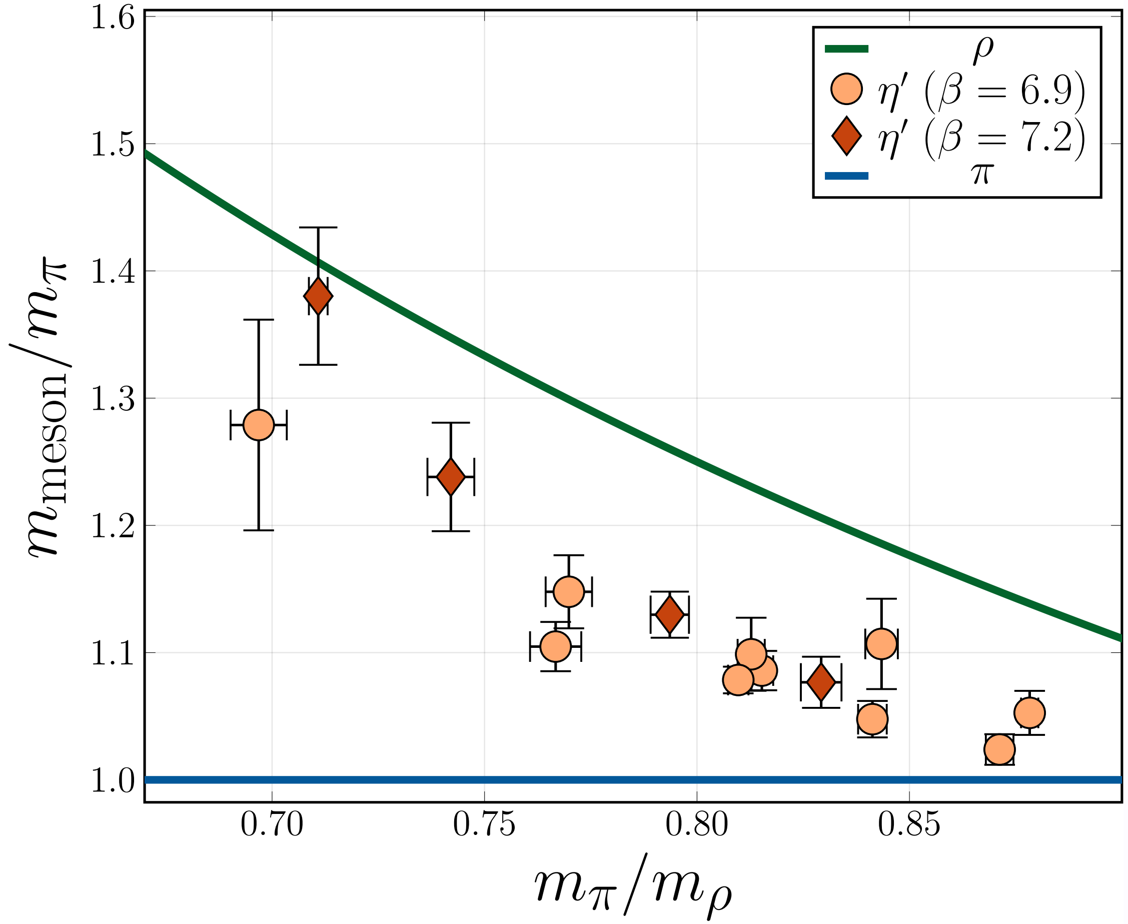
Bennett et. al. [1909.12662]



The pseudoscalar and vector mesons are the lightest non-singlets.

[1] Bennett et. al. [2304.07191]

The pseudoscalar singlet η' is surprisingly light!



- Phenomenologically relevant:
  - $m_\rho > m_{\eta'}$ different from QCD
  - relevant low-energy dof
  - η' relevant for $\pi\pi$ scattering
  - more accessible channels for decays into SM

Interesting! Is this surprising?

Bennett et. al. [2304.07191]

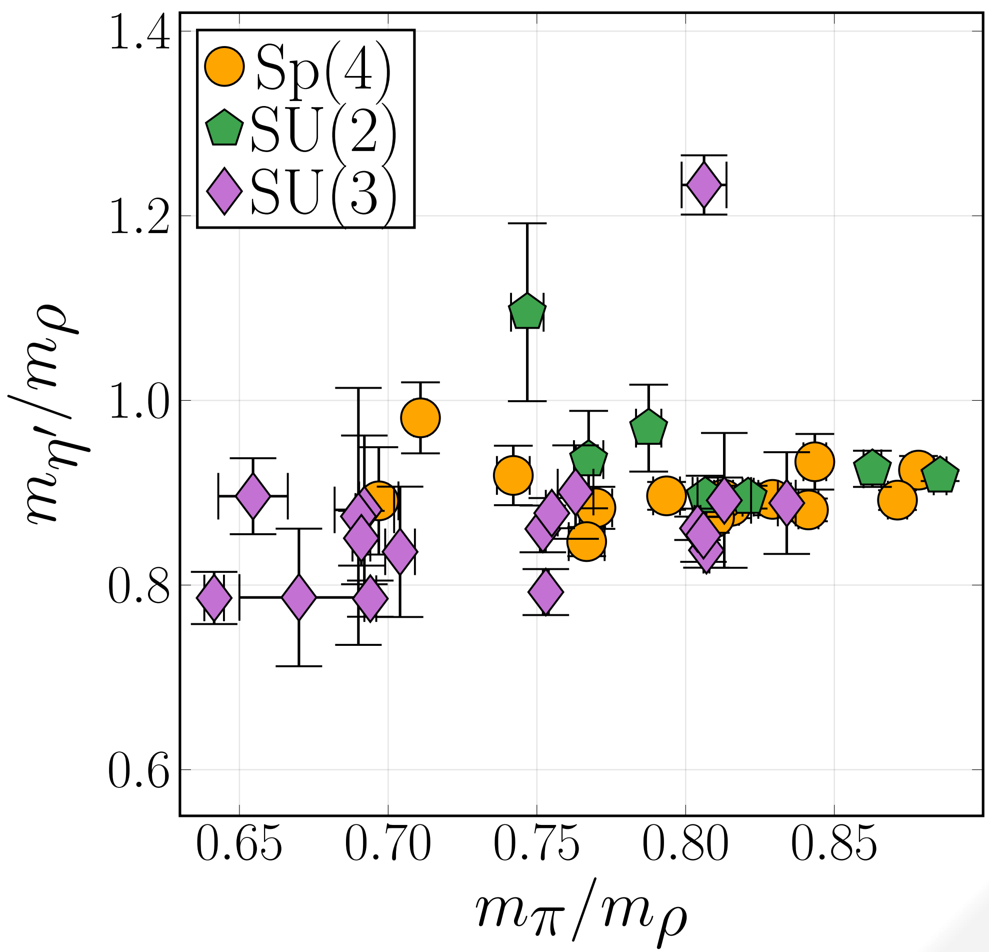
Consider different theories:

- Large N_c: $m_{\eta'} - m_\pi \propto N_f / N_c$
  - $N_f = 2$ could be "small"
  - $N_c = 4$ could be "large"

SU(2) and SU(3) comparison:

- Similarities:generic $N_f = 2$ feature?
- QCD: strong N_f dependence
- Differences may arise $m_\pi / m_\rho \to 0$

mass driven by flavour content!



Kulkarni al. [2202.05191]

Consequences for Dark Matter

- Mass hierarchies: limit χPT validity
  - inclusion of other states than π required, e.g. η' and ρ
  - additional tests needed (fermions are too heavy)
- Light unprotected states η', π^0 allow decays into SM
  - no protection from symmetry

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Are these fermion masses phenomenologically relevant?
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Dark Matter Scattering on the Lattice

- Pions are in the 5-dimensional representations
- A two pion scattering is in one of three irreps

$$5 \times 5 = 14 \oplus 10 \oplus 1$$

- Corresponds to the usual QCD channels
  - $14 \Leftrightarrow$ isospin $I = 2$ in QCD, e.g. $\pi^+\pi^+$
  - $10 \Leftrightarrow$ isospin $I = 1$ in QCD, e.g. $\pi\pi \to \rho$
  - $0 \Leftrightarrow$ isospin $I = 0$ in QCD, e.g. $\pi\pi \to \sigma/f_0$

[1] M. Lüscher (Nucl.Phys.B 1991)

Scattering information from the lattice

- Scattering phase shift $\delta_0(p)$ from finite volume energy

$$\tan(\delta_0(q)) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}, \quad q = p^* \frac{L}{2\pi}$$

$$\cosh\left(\frac{E_{\pi\pi}}{2}\right) = \cosh(m_{\pi\pi}) + 2\sin\left(\frac{p^*}{2}\right)^2$$

- Low-velocity behaviour: Scattering length

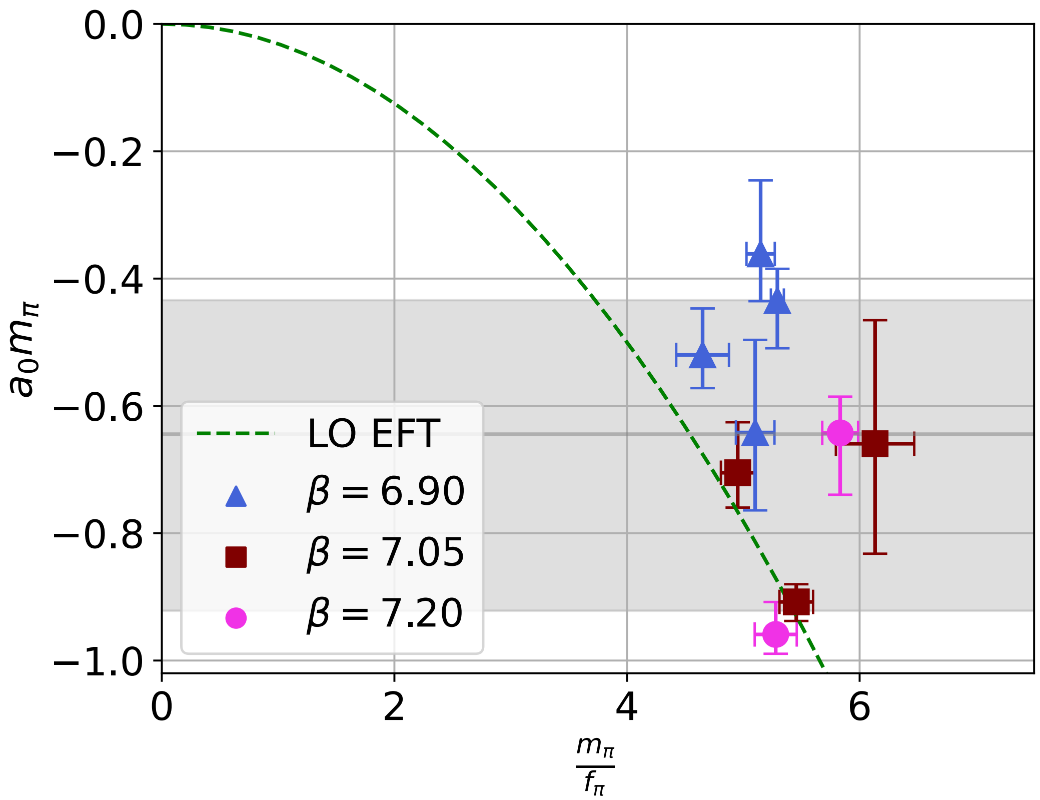
$\Rightarrow$ relation between $\pi\pi$ energy $E_{\pi\pi}$ and m_π on a lattice [1]

$$\frac{\delta E_{\pi\pi}}{m_\pi} = \frac{4\pi m_\pi a_0}{(m_\pi L)^3}\left(1 + c_1 \frac{m_\pi a_0}{m_\pi L} + c_2 \left(\frac{m_\pi a_0}{m_\pi L}\right)^2\right)$$

[1] Dengler et.al. [2311.18549]

see also Arthur et. al. [1412.4771] for SU(2)

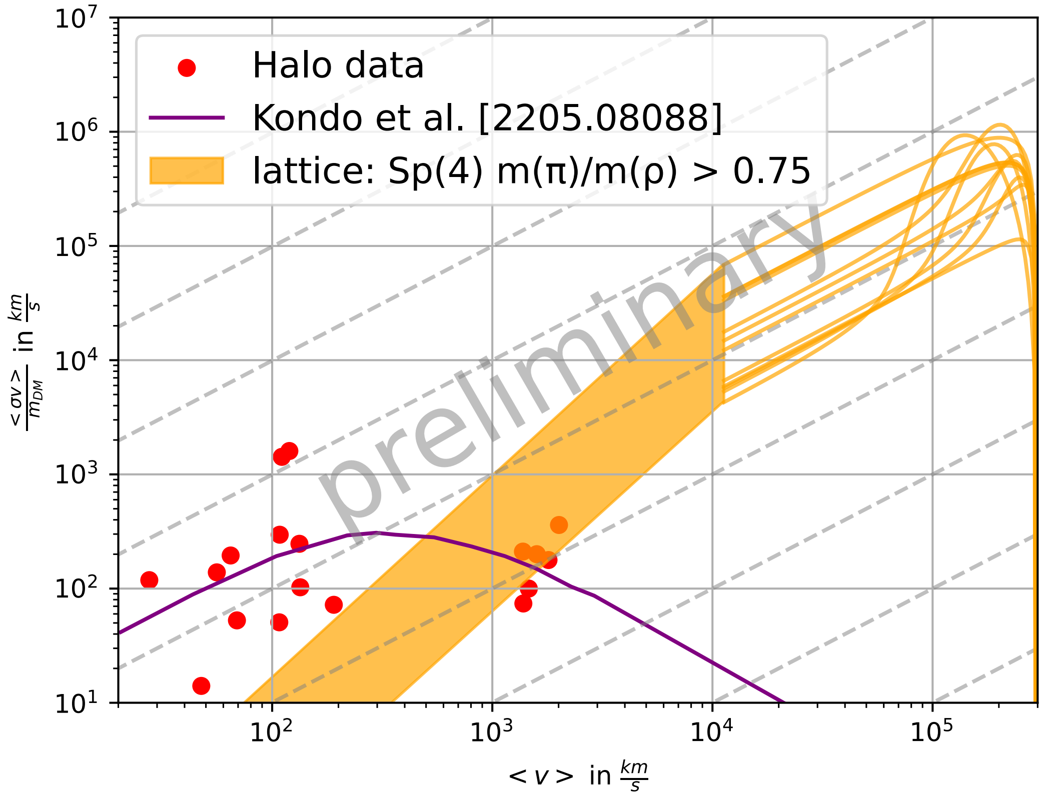
Blum et.al. [2301.09286] for SU(3)



First investigation of isospin-2 scattering

- repulsive $\pi\pi$ interaction
- few lattice energy levels available $\Rightarrow$ systematics
- finite volume effects present
- roughly matches ChiPT

Plot: courtesy of Yannick Dengler



First investigation of isospin-2 scattering

- phase shift $\delta(p)$ gives velocity dependence $\langle \sigma v \rangle$
- No velocity dependence in isospin-2 channel
- Overall scale chosen to match low velocity behaviour

Summary

- Full light hadron spectrum of two-flavour $Sp(4)$
  - surprisingly light η'
  - input for EFTs: masses and decay constants
  - first determination of isospin-2 $\pi\pi$ scattering

Outlook

- Full scattering analysis of $2\pi \to 2\pi$ and $3\pi \to 2\pi$
  - velocity dependence from strong resonances?
- Better understanding of singlets and scattering states
- Singlet spectroscopy closer to the chiral limit

Thank you